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Edited for the Council by J. LAURENCE PRITCHARD, Fellow

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NOTICES

President

The Council have accepted with very great regret the resignation of Lord Weir, President of the Society.

Chairman Elect

At a meeting of Council held on June 8th, Major-General Sir W. Sefton Brancker, K.C.B., A.F.C., was elected Chairman for the period from October, 1925, to September, 1926.

Elections

The following Members were elected at a Council Meeting held on June 26th:

Fellow.—Major-General Sir W. Sefton Brancker, K.C.B., A.F.C.
Associate Fellows.—Mr. R. A. Logan and Mr. G. J. Mead.
Student.—Mr. W. J. Bernhard.
Associate Member.—Squadron Leader W. G. Sitwell, D.S.O.
Foreign Member.—Mr. Chang S-Lin.

R.38 Memorial

The Memorial to those lost in the disaster to R.38 (Z.R.2) which has been placed in the Society's Library was unveiled by the American Ambassador on June 29th. An account of the ceremony will be found on page 296.

Broadcasting

The attention of members is called to the fact that Colonel the Master of Sempill will give a talk on the Society from the London Broadcasting Station on July 24th.

Library

The following books have recently been received and placed in the Library:—"Das Zeppelinluftschiff," F. Kollmann; "The Complete Airman," G. C. Bailey; "Taschenbuch der Luftflotten 1924/25," W. von Langsdorff; Faksimile-Drucke des Thuringer Staatsarchiv Heft I "Die Flugseug Handschrift" des Melchio Bauer von 1764; Technical Report for 1921-22, and 1922-23, of the Advisory Committee for Aeronautics; "Mon Raid," Pelletier Doisy; "The Rigid Airship," E. H. Lewitt; "The Metallurgy of Aluminium and Aluminium Alloys," R. J. Anderson; "An Introduction to Fluid Motion," W. N. Bond; "La Legitimité de la Guerre Aerienne," A. Henry-Coüannier; "The Resistance of Express Trains," C. F. Dendy Marshall; and the Ninth Annual Report of the National Advisory Committee for Aeronautics, 1923.

Associate Fellowship Examination

Provided that sufficient entries are received, the Society's Examination for Associate Fellowship will be held on Monday, September 21st (Part I.), and Tuesday, September 22nd (Part II.). Entry forms may be obtained from the Honorary Secretary, and must be returned duly completed not later than Monday, August 24th.

Annual Holidays

The Offices of the Society will be closed from August 1st to 10th inclusive.

J. LAURENCE PRITCHARD, *Hon. Secretary.*

XUM

R.38 MEMORIAL

The tablet to the memory of the British and Americans who lost their lives in the disaster to the airship R.38 over the Humber on August 24th, 1921, has been placed in the Library of the Royal Aeronautical Society, Albemarle Street, W., and was unveiled by the American Ambassador, Mr. Alanson B. Houghton, on June 29th.

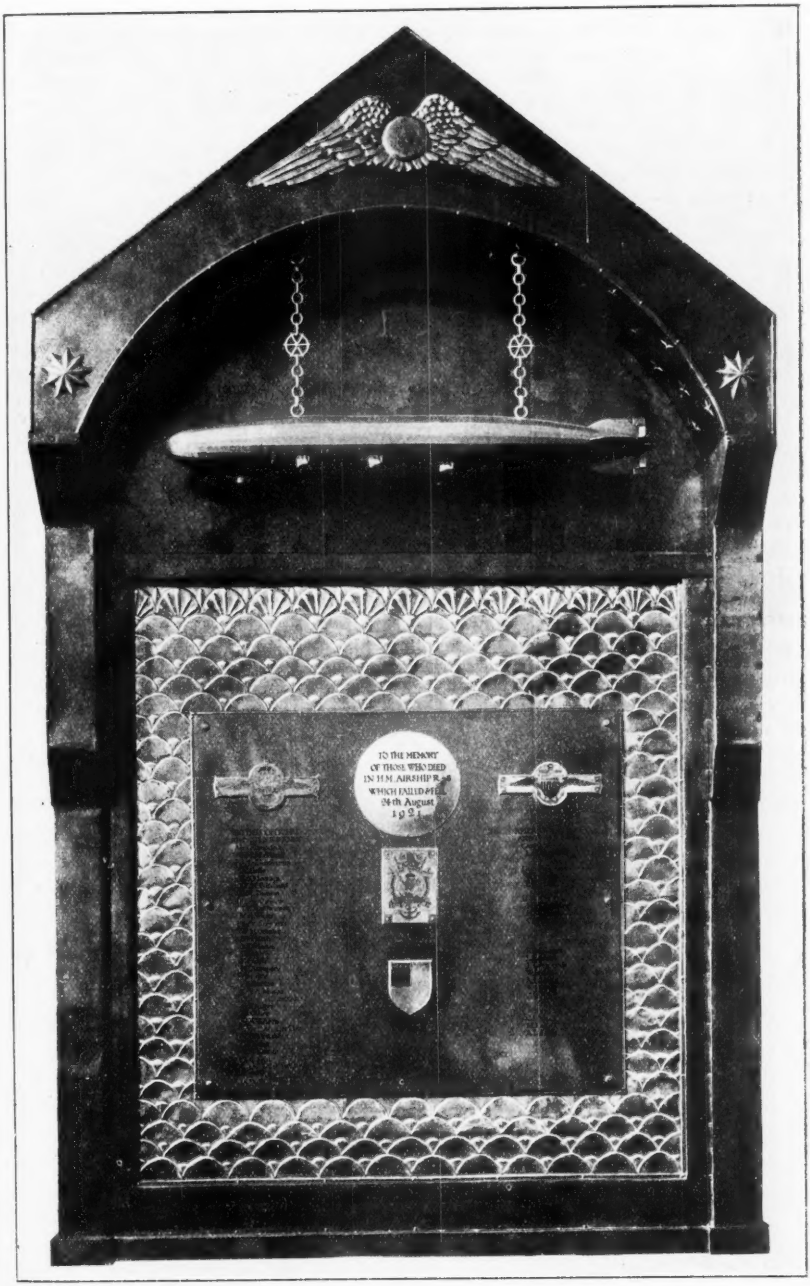
Only four of the occupants were saved, and two of these, Mr. H. Bateman, of the National Physical Laboratory, and Corporal W. A. Potter, were present at the ceremony. About thirty of the relatives of those who lost their lives were also present. At the time of the disaster the airship, which had been sold to the United States Navy, was manned by a mixed crew of Royal Air Force and United States Navy officers and men. Among those who lost their lives were: Air Commodore E. M. Maitland, Flight Lieutenant J. E. M. Pritchard, Flight Lieutenant Thomas, Mr. C. I. R. Campbell (chief designer), Mr. J. R. Pannell, and Mr. C. W. Duffield (of the National Physical Laboratory), and Commander L. A. H. Maxfield, Lieutenant-Commander W. N. Bieg, and Lieutenant-Commander E. W. Coil, of the United States Navy. A full list of those who lost their lives appeared in the Journal for September, 1921.

The airship was originally designed in 1918 for war purposes with the British Navy, and was completed in June, 1921. When it was decided to close down all experimenting with airships she was sold to the United States Navy and renamed "Z.R.2." On her fourth and last flight on August 24th, 1921, when she was manned by a mixed crew of Royal Air Force and United States Navy officers and men, and representatives of the design and research staffs at the National Physical Laboratory, she broke her back and fell into the Humber, and only four of the occupants were saved. The Council of the Royal Aeronautical Society, viewing with concern the stoppage of British airship development, decided to raise a fund with the dual purpose of providing a memorial tablet in its library and of stimulating the continuance of interest in airship research and progress. The tablet, which has been designed by Mr. Paul Cooper, records the names of those lost, under the shields of the Royal Air Force, United States Navy, Institute of British Naval Architects, and National Physical Laboratory.

Lieutenant-Colonel H. T. TIZARD (Chairman of the Society), in asking the American Ambassador to unveil the memorial, said the loss of the R.38 seemed to destroy the wavering faith of many people in airships. The faith of the members of that Society, however, remained unshaken, and they decided to raise a fund with the dual purpose of providing a memorial tablet in the Library of the Society (which had numbered several of the crew among its members) and of stimulating the continuance of interest in airship research and progress. Public opinion was already swinging back, and a small body of experts had been entrusted with the task of regaining all, and more than all, that was lost in the R.38.

The American Ambassador then unveiled the memorial, but did not make any remarks.

Professor LEONARD BAIRSTOW, Zaharoff Professor of Aeronautics, University of London, in thanking the American Ambassador for unveiling the memorial, said the men of the R.38 had the courage of their belief in the future of airships, and the failure of the ship was a misfortune to development, both in the loss of knowledge possessed by those American and British pioneers and in the seal it set on an earlier decision to discontinue airship development in Britain. The



R.38 Memorial.

airship would, in his conception, be an addition to existing facilities, an addition which would make possible things not yet dreamed of, say, for example, 24 hours from the limits of civilisation to the North Pole or 100 hours from London to India. The handling of airships was now greatly different from that when R.38 was designed, and in the use of mooring masts they saw a further marked advance towards safety. He had not forgotten the facts that in America the Shenandoah broke away from the mast, and that recently R.33 was torn from her mooring at Pulham; but in both cases the airships, under magnificent airmanship, made good and returned to their bases. The airmen of those ships were the successors of the men they honoured that day. Perhaps the tale of misfortune was not all told, but he believed that the well-being of the world was being served by the enthusiasts who devoted their energies to the development of airships. As representing British scientific aeronautics, he expressed the hope that progress would be based on knowledge, *i.e.*, research, as part of the tribute which the nations owed to those who had gone before.

Among those present were:—Admiral McNamee (United States Navy), Air Vice-Marshal Sir Sefton Brancker (Director of Civil Aviation and Chairman-elect of the Council of the Society), Major Davidson (United States Embassy's Military Staff), and two of the survivors (H. Bateman, representative of the National Physical Laboratory, and Corporal W. A. Potter), Commander J. C. Hunsaker and Commander Towers (United States Embassy's Naval Staff), Sir Eustace Tennyson-d'Eyncourt, Sir Malcolm Fraser, Major C. H. Scott (who piloted R.34 to America and back), Sir Philip Sassoon, M.P. (Under-Secretary of State for Air), Sir John Shelley Rolls and the Hon. Lady Shelley Rolls, Members of the Council of the Royal Aeronautical Society, and Lieutenant-Colonel W. Lockwood Marsh.

WILBUR WRIGHT LECTURE, APRIL 30TH, 1925

The Wilbur Wright lecture by Rear-Admiral D. W. Taylor, Chief Constructor, U.S. Navy, retired, was read by Commander J. C. Hunsaker, U.S.N. (C.C.), in the Society's Library at No. 7, Albemarle Street, London, W.1, on April 30th, 1925. The chair was taken by Lieutenant-Colonel H. T. Tizard, Chairman of the Society, who opened the proceedings by explaining that Admiral Taylor had unfortunately been compelled through illness to return to America before giving the Wilbur Wright lecture, and that Commander Hunsaker had very kindly agreed to read the lecture for him. Before calling upon Commander Hunsaker, the Chairman said:—

I think the audience would like to hear something of Admiral Taylor's career. After graduating from the U.S. Naval Academy he was sent for professional training to the Royal Naval College, Greenwich. During his residence in this country he became interested in the work of the Froudes on ship resistance. This he later applied in the Washington model tank, which he built and directed for many years.

Admiral Taylor's researches on the general problem of ship propulsion and resistance are now a classic and are to be found in his book, "Speed and Power of Ships." In his reduction of ship's water lines to mathematical form, using a souce-sink distribution, he anticipated the modern aerodynamic treatment of the problem and for it was awarded the gold medal of the Institution of Naval Architects of Great Britain.

As Chief Constructor of the U.S. Navy he was responsible for the design of both warships and aircraft, which he placed upon a basis of applied scientific research. As early as 1912 Admiral Taylor foresaw the transition of the aeroplane from the hands of inventors into those of engineers and built an 8-foot wind tunnel with the maximum speed of 70 miles an hour at his model basin in order to obtain the fundamental data. This tunnel is still in constant use.

Under Admiral Taylor's energetic leadership the U.S. Navy developed types of naval aircraft, including both non-rigid and rigid airships to suit its needs, and then adapted the warships to carry seaplanes by providing them with catapults.

Admiral Taylor's experience has been unique in that he has maintained his position as one of the greatest naval architects and shipbuilders of modern times, while at the same time he has played a leading part in aeronautical engineering development. As Chief Constructor his aircraft programme was consistently based upon research and experiment.

Since his retirement, Admiral Taylor has devoted his time to the direction of the research activities of the National Advisory Committee for Aeronautics, and the present paper discusses some recent experimental results.

SOME ASPECTS OF THE COMPARISON OF MODEL AND FULL SCALE TESTS.

BY D. W. TAYLOR, E.D., D.SC., LL.D.,

MEMBER AND SECRETARY, U.S. NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Aeronautics now covers a large field. The bibliography alone, compiled and published annually by the United States National Advisory Committee for Aeronautics, requires something like two hundred pages of a book seven inches by ten inches. Needless to say, I am not undertaking to review the whole field.

Owing to the difficulties of conducting free flight tests of performance and the fact that we cannot afford to make many mistakes in an appliance whose operation involves the risk of human life, it is peculiarly desirable that we may be able to predict the performance of the completed airplane from small-scale experiments; and probably in no other branch of mechanical science have we at present so many research laboratories.

In view, then, of the universal use of models and wind tunnel tests to obtain results upon which are based predictions of performance of full-sized airplanes, it appears worth while to give some consideration to the foundation, as it were, of such methods. The mathematical basis of the law of mechanical similitude has been traced back as far as Sir Isaac Newton, but it is believed that the first serious practical application was that made by Mr. William Froude, when, some sixty-five years ago, with the aid of the Admiralty, he built in his garden at Torquay a long tank filled with water, in which he tested models of vessels. Froude's methods have been universally accepted by naval architects as of great value, and they are able to predict performances of full-sized ships with accuracy adequate to the purposes of the engineer. Nevertheless, they are not exact, and in the last analysis their justification is due to the fact that the results they predict for the full-sized ship are substantially verified in practice. However, Froude separated the frictional resistance of the model from its wave-making resistance, or the resistance absorbed in the production of waves, and it is to the latter only that Froude's Law of Comparison applies. Frictional resistance is calculated from coefficients originally determined by Froude upon the basis of tests with comparatively small plane surfaces at low speeds, and it is generally recognised now by naval architects that large-scale experiments would be desirable to give us greater assurance of accuracy when dealing with present day ships.

The most fundamental and instructive method of covering this whole question of the value of model experiments is based upon the principle of dimensional homogeneity first fully enunciated, I believe, some fifteen years ago by a Russian, Riabouchinski. In the United States Dr. Buckingham has taken up the matter and done much work to amplify, clarify, and apply the principle. In a paper in 1915, before the American Society of Mechanical Engineers, he gave a number of illuminating applications. In the mathematical treatment below I follow essentially Buckingham's methods.

Instead of considering the general formula, which may be of a beautiful simplicity to the mathematical physicist, but is not too easy to follow for us who are not mathematical physicists, I will consider only the general case applying to motion of objects in a fluid medium. The first thing to establish is the quantities involved, that is, the physical quantities present which can affect the case. Let us denote by R the resistance of the object; by V its speed; by L its size, or some linear dimension; by ρ the density of the fluid; by μ the viscosity of the fluid; by C the compressibility of the fluid; and by g the acceleration of

gravity. In addition, there are certain ratios present which I will denote by r_1, r_2, etc. These ratios express certain physical facts, such as aspect ratio of an airfoil, its angle of attack, etc. They are all independent of size.

It may be that the quantities enumerated above do not comprise all of the quantities we should consider, but they do comprise the most obvious ones and are sufficient in number to illustrate the point desired to be made. Now all these quantities can be expressed in terms of three units, and we will choose the simplest and most commonly used units, namely, those of mass, m; length, l; and time, t. Each of these physical quantities also has well-known dimensions. The table below gives the quantities enumerated above and their dimensions in m, l and t.

Quantities involved.	Dimensions in m, l and t.
R Resistance	$m\ l\ t^{-2}$
V Speed	$l\ t^{-1}$
L Size	l
ρ Density of fluid	$m\ l^{-3}$
μ Viscosity of fluid	$m\ l^{-1}\ t^{-1}$
C Compressibility of fluid	$m^{-1}\ l^{-1}\ t^{2}$
g Acceleration of gravity	$l\ t^{-2}$
r_1, r_2, etc., Ratios	Dimensionless

Now if the quantities above have a relation connecting them, it may be written as follows:—

$$F(L, \rho, V, R, \mu, C, g, r_1, r_2 \ldots) = 0 \quad . \quad . \quad . \quad (1)$$

This equation, of course, teaches us nothing except that there is some relation between the seven physical quantities entering the case. Now let us choose three of the above quantities (this is because we have three units to express them all) and, instead of writing the relation symbolically between the seven simple quantities, let us use the following, involving the seven quantities in four compound quantities or variables:—

$$F(L^a \rho^b V^c R,\ L^d \rho^e V^f \mu,\ L^g \rho^h V^k C,\ L^l \rho^m V^n g,\ r_1,\ r_2 \ldots) = 0 \quad . \quad (2)$$

By the principle of dimensional homogeneity, since the physical relations or facts expressed by the above do not change with change of units, the compound variables or quantities above must be dimensionless, that is, of zero dimensions. By expressing their dimensions in terms of the dimensions of the three fundamental units, we have for each quantity three equations to determine the exponents a, b, c, etc. Let us take the first quantity. Our dimensional equation is

$$L^a \rho^b V^c R = l^a\ m^b\ l^{-3b}\ l^c\ t^{-c}\ m\ l\ t^{-2}$$
$$= l^{a-3b+c+1}\ m^{b+1}\ t^{-c-2}$$

In order that the expression may be dimensionless, we must have the index of l, for instance, equal to zero; that is, $a - 3b + c + 1$ equal to zero; similarly, the indices of m and t must equal zero. This gives us the three equations below, whose solution is obvious:—

$$\left.\begin{aligned} a - 3b + c + 1 &= 0 \\ b + 1 &= 0 \\ -c - 2 &= 0 \end{aligned}\right\} \text{whence} \quad \begin{aligned} a &= -2 \\ b &= -1 \\ c &= -2 \end{aligned}$$

So our first quantity is $R / \rho L^2 V^2$.

Proceeding to the second quantity and treating it in exactly the same manner, we have the following:—

$$L^d \rho^e V^f \mu = l^d\ m^e\ l^{-3e}\ l^f\ t^{-f}\ m\ l^{-1}\ t^{-1}$$
$$= l^{d-3e+f-1}\ m^{e+1}\ t^{-f-1}$$

Whence

$$\begin{aligned} d-3e+f-1&=0 \qquad & d&=-1\\ e+1&=0 \qquad & e&=-1\\ -f-1&=0 \qquad & f&=-1 \end{aligned}$$

Then our second expression is $\mu/L\rho V$. Of course, in practice it makes no difference whether we use the above expression or its reciprocal. The ratio μ/ρ is a quantity a good deal used in physical parlance, called the kinematic viscosity, and denoted by ν. So our second variable will be ν/LV.

Proceeding in just the same manner for the third and fourth quantities we finally reduce the general equation to

$$F_1\left(\frac{R}{\rho L^2V^2}\ \frac{\nu}{LV}\ \rho L^2V^2C\ \frac{Lg}{V^2},\ r_1,\ r_2\ldots\right)=0 \qquad . \quad . \quad (3)$$

Now ρL^2C has of course the dimensions of $1/V^2$ and since the velocity of sound in air is proportional to $1/\sqrt{\rho C}$ if we denote by V_s the velocity of sound in air we may use instead of the variable ρL^2V^2C the variable V^2/V_s^2.

Now we can solve the above symbolically for any one of the compound variables. Solving for the first, we have

$$\frac{R}{\rho L^2V^2}=F_2\left(\frac{\nu}{LV},\ \frac{V^2}{V_s^2}\ \frac{Lg}{V^2},\ r_1,\ r_2\right)$$

Since we are interested primarily in the resistance, R, let us transform the above as below:—

$$R=\rho L^2V^2F_3\left(\frac{\nu}{LV},\ \frac{V^2}{V_s^2},\ \frac{Lg}{V^2},\ r_1,\ r_2\ldots\right) \qquad . \quad . \quad . \quad (4)$$

The ratios r_1, r_2, etc., express such things as aspect ratio, angle of attack, etc., and hence are obviously the same for the model as for the full-sized object, so that for purposes of comparison between model and full-sized object they affect the case only as constants or fixed coefficients, to be determined by experiment or some other independent method, and can be eliminated from the equation above. This reduces us finally to the general equation:—

$$R=\rho L^2V^2F_3\left(\frac{\nu}{LV}\ \frac{V^2}{V_s^2}\ \frac{Lg}{V^2}\right) \qquad . \quad . \quad . \quad . \quad (5)$$

This, then, is a relation which follows if all of the factors which we originally assumed enter into the case and affect our results. We do not know whether, as a matter of fact, all these factors do affect them as indicated by the general expression for R above. But, obviously, if all the factors enumerated materially affect our results, producing the preceding equation (5), model experiments are of no value for predicting the performance of the full-sized object. In the model experiment we make an object differing in scale from the full-sized object, and test it at a speed different from that of the full-sized object. If this method is to be of value in practice, as a general thing models should be smaller and tested at a lower speed than for the full-sized object. From equation (5) above, however, we see, considering the first combined variable, which we must have constant in proceeding from model to full-sized object, that LV must be constant, since we know that ν is practically constant for standard air. Considering the second term, however, since the velocity of sound in air is constant, if V^2/V_s^2 is to be constant V must be constant. Considering the third term, g is constant, and if Lg/V^2 is to be constant we must have L/V^2 constant. These three requirements evidently reduce to the single one, namely, that neither L nor V can change. In other words, we cannot use the model and obtain results for the full-sized object.

However, the equation (5) does not necessarily apply to the case unless it is confirmed by theoretical demonstration, experience, or practical tests. We do not know that, as a matter of fact, the compound quantities which we originally

considered as possibly affecting the case do all affect it. Suppose none of these quantities has any effect. We then have the exceedingly simple formula

$$R = \rho L^2 V^2 \times \text{a coefficient.}$$

If this expresses the facts, a single experiment at a single speed of a model gives us complete information on the resistance at all speeds for all sizes of similar objects.

There is a theoretical basis for regarding this as a basic expression for resistance, the departures from it being, if appreciable, of secondary importance. If the disturbance of a fluid by an object moving through it, or what is simpler to grasp, if the lines of flow of a fluid past a submerged object, do not change with speed then all forces vary as the square of the speed. For any force is measured by the momentum generated in unit time in the opposite direction and, taking momentum at such a distance that pressure is not affected, the momentum generated in unit time is proportional to the square of the velocity. Similarly, if the lines of flow are similar as we change size, the momentum generated must vary as L^2. From consideration of a perfect—non-viscous—fluid we reach similar conclusions, but, as it happens, in a theoretical perfect fluid objects have theoretically no resistance. Concluding, then, that the expression $R = \rho L^2 V^2 \times$ a coefficient is a correct first approximation, let us see what we can do to reach a closer approximation.

Suppose that only one of the terms of F_3 in (5) is significant, the rest having no bearing. If the first term is the only significant one, we have

$$R = \rho L^2 V^2 F_3 (\nu / LV)$$

Similarly, if the second and third terms are the only significant ones, we have

$$R = \rho L^2 V^2 F_3 (V^2 / V_s^2)$$

or

$$R = \rho L^2 V^2 F_3 (Lg / V^2)$$

Now it is obvious that if only one term is significant, we may or may not have a possible basis for model experiment, depending upon the nature of the term. Consider first the expression

$$R = \rho L^2 V^2 F_3 (\nu / LV)$$

Here the requirement is that ν / LV should be constant, or, what is the same thing, that LV / ν should be constant. This results in the undesirable condition that if ν is constant LV must be constant as we pass from model to full-sized object, so that the smaller the model the higher the speed at which it must be tested. This is not a very desirable condition.

Consider now the second requirement:—

$$R = \rho L^2 V^2 F_3 (V^2 / V_s^2)$$

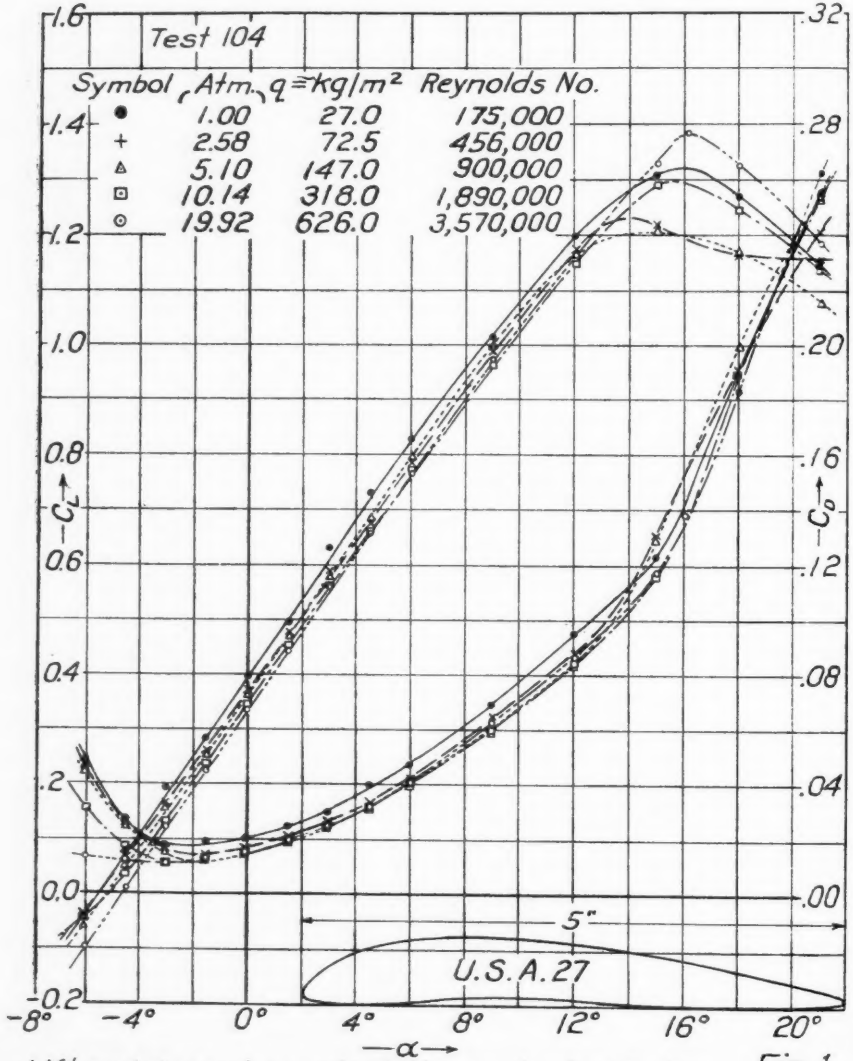
If V^2 / V_s^2 is constant, since V_s itself is constant, V must be constant, and we cannot use a low-speed model.

Consider now the third condition. Here our requirement is that Lg / V^2 is constant. This is the form found so useful in testing ships' models. The relation that speeds shall be as the square root of linear dimensions results in the test speed for the model being low, so that tests can be easily made. Evidently, however, for a body completely submerged in and surrounded by a fluid, the action of gravity can have practically no effect until the proposed speeds approach the point where vacua are formed in the fluid. Hence we can confidently eliminate from our general equation above the variable Lg / V^2 as the one to govern our second approximation.

Consider next the variable V^2 / V_s^2. Practically all the speeds with which we are concerned in airplane work, except some propeller speeds, are far below the velocity of sound through air, and there is little reason to believe that the compressibility of the air has a material effect because the compression is so small.

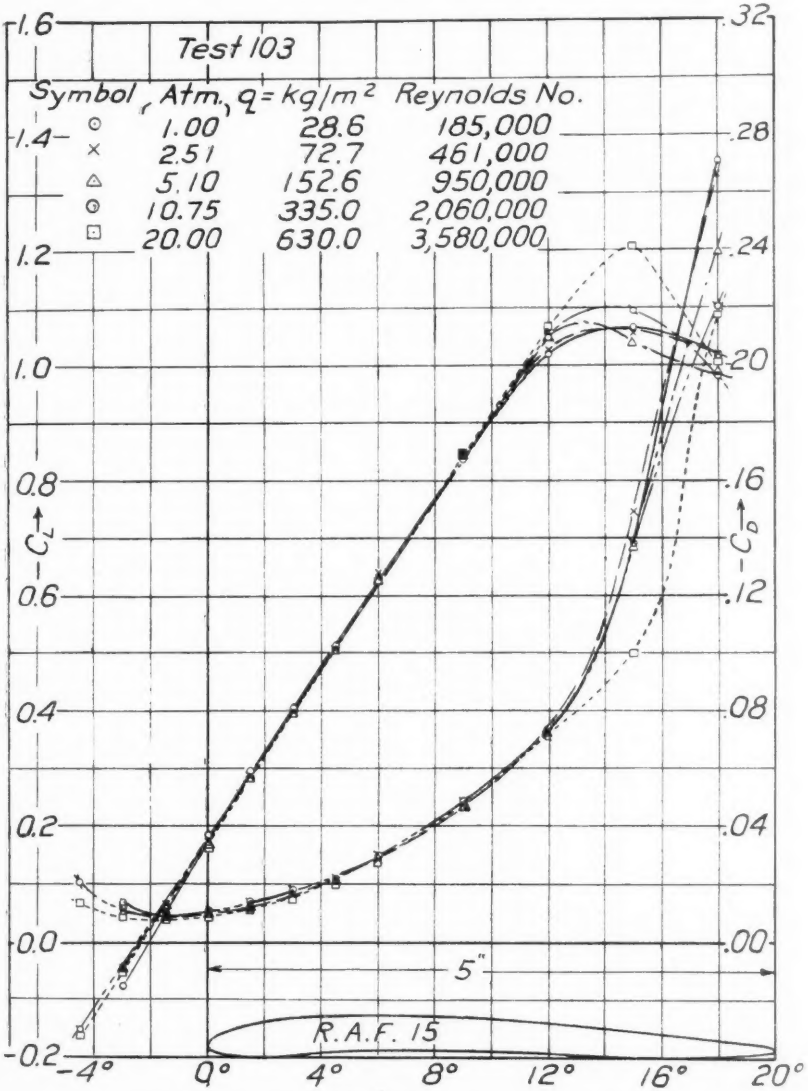
Also, experiments with projectiles indicate the same conclusion. Hence we can eliminate the second term as the one that we must keep when we seek a second approximation. When we come to the first term, however, the case is different. We know that air has viscosity, and we know that the viscosity must have some action at all speeds. Hence the first term cannot be argued away on general principles. Also it may be remarked in passing that the expression

$$R = \rho L^2 V^2 F_3 (\nu / LV)$$



Lift and drag characteristics as tested in the Variable Density Wind Tunnel from 1 to 19.92 atmospheres. Fig. 1

can be derived independently of considerations of dimensional homogeneity from the equations of motion of a viscous fluid. However, these equations of motion necessarily assume in the first place that there is no other factor, such as compressibility, gravity, etc., involved. We might have originally assumed some more physical quantities present and affecting matters such as nature of surface,



Lift and drag characteristics as tested in the Variable Density Wind Tunnel from 1 to 20.0 atmospheres Fig. 2

or sizes of turbulent vortices in the wind tunnel, but we seem to be restricted to one variable in our F_3 function if we are to profit by model experiments and the viscosity variable seems the one we should choose. The wisdom, or otherwise, of the choice will be shown if model experiments in accordance with the formula do or do not predict full-scale performance.

Having then reduced our original broad formula to

$$R = \rho L^2 V^2 F_3 (LV/\nu) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

involving the density, the size, the speed, and some unknown function of LV/ν, the well-known Reynolds' number, we need to form some conception of the effect of Reynolds' number, commonly called the scale effect. While we do not know the form of the function, we do know for the flow of water, oil, and air in pipes the relative experimental values. The original wonderful experiments by Reynolds have been repeated and amplified by others since 1880, and it seems established that at low speeds where the fluid flows smoothly F_3 has one set of values, and at high speeds when the motion is completely turbulent there is another well-defined set of values, while for intermediate speeds values are rather indeterminate. Wind tunnel investigations on such objects as cylindrical wires, struts and streamline wires show that the resistance departs appreciably from the Law of the Square with variation of Reynolds' number.

When we come to such objects as an airplane, however, we have difficulty with the ordinary wind tunnel. For constant Reynolds' number to test a model, say one-twentieth scale, would require wind tunnel speed twenty times the actual flying speed, and there are no wind tunnels that can come in sight of this performance. Such speeds would be greater than the velocity of sound. There appears to be only one practicable solution of the difficulty, namely, the use of a testing tunnel where we vary the density of the air and hence the value of ν.

The kinematic viscosity coefficient ν for air varies inversely as the pressure and decreases with temperature according to a somewhat complicated relation. Table I. below gives numerical values when the unit of length is the centimetre and the unit of time the second.

TABLE I.

Kinematic Viscosity Coefficients ν in cm²/sec.

Pressure in atmospheres. Temp. Cent.	1/10.	1.	5.	10.	20.
50	1.284	0.1284	0.02568	0.01284	0.00642
40	1.292	0.1292	0.02584	0.01292	0.00646
30	1.300	0.1300	0.02600	0.01300	0.00650
20	1.308	0.1308	0.02616	0.01308	0.00654
10	1.318	0.1318	0.02636	0.01318	0.00659
0	1.329	0.1329	0.02658	0.01329	0.00665
−10	1.340	0.1340	0.02680	0.01340	0.00670
−20	1.351	0.1351	0.02702	0.01351	0.00676
−30	1.364	0.1364	0.02728	0.01364	0.00682
−40	1.378	0.1378	0.02756	0.01378	0.00689
−50	1.392	0.1392	0.02784	0.01392	0.00696

The variable density wind tunnel of the National Advisory Committee for Aeronautics, as originally suggested by Dr. Munk of our staff, was described to the Society two years ago and a few sample results given. A good deal of experience has been had since then with the appliance. One lesson of experience has been that when we are working under a pressure of twenty atmospheres it takes but a small electrical spark to kindle a substantial fire. However, these little practical difficulties have been overcome, and experience in testing a number

of different airfoils, etc., indicates that this apparatus, or the equivalent, is essential if we are to make a thoroughly reliable second approximation to the performance of an airplane from model tests.

Reynolds' number—LV/ν—is a compound ratio whose numerical value in the case of any given object depends upon the ratios between the actual values of LV and ν and their unit values. Unfortunately, each type of object has its own series of Reynolds' numbers, because as a rule the values of L are not comparable for dissimilar objects. Thus, for an airplane wing we naturally use for L in Reynolds' number the length of the chord. For an airship we would use the length or the diameter or any linear function of the two. But L for the airship would not be comparable with L for the airplane.

Considering airplanes as they are using the chord of the wing in inches as L, and speeds in statute miles per hour, the Reynolds' numbers come out fairly large. Thus, for an airplane of 5 foot chord, at 100 miles per hour in a normal atmosphere the Reynolds' number will be some 4,800,000. For its model of 6in. chord in a wind tunnel at 100 miles per hour with normal air the Reynolds' number will be 480,000.

Attention is invited now to Figs. 1 to 3, giving in condensed form results of recent tests of three airfoils of well-known form in the variable density wind tunnel. Necessary data as to the conditions and the airfoil section to which they apply are shown on each figure. Results are plotted as curves of lift and drag coefficients as ordinates over angles of attack—α—as abscissæ, following the standard practice of the U.S. Advisory Committee. Fig. 1 shows results for an American section, U.S.A. 27; Fig. 2 shows results for a British section, R.A.F. 15; Fig. 3 shows results for a German section, Göttingen 387. It happens that these three typify the medium, the thin and the thick sections.

Ignoring minor eccentricities due to accidental causes, unavoidable experimental error, etc., these curves seem to warrant a few broad conclusions which, by the way, are in agreement with other results too numerous to include.

In the first place, the scale effect appears to have more influence upon the drag than upon the lift. This may be explained upon theoretical grounds.

In the second place, the scale effect increases more and more slowly as the Reynolds' number increases so that conclusions drawn from experiments with airfoils within the Reynolds' number range of ordinary wind tunnels cannot safely be extended to much larger Reynolds' numbers.

In the third place, the consistency of the results gives us reason to think that for present day airplanes we are justified in ignoring the effect of other factors than Reynolds' number in reaching our second approximation to aerodynamic properties of airfoils.

In the fourth place, the thin airfoil appears to show less scale effect than the thick airfoil.

In the fifth place, so far as airfoil action is concerned, the scale effect is, after all, secondary, though by no means negligible when we undertake to estimate closely.

The comments above apply only to airfoils. They do not necessarily apply to wires, struts, etc. Such appendages can be tested separately in the ordinary wind tunnel at Reynolds' numbers much closer to the numbers on the full-sized airplane than is possible with the airplane structure proper.

When we enter the somewhat vexed field of aircraft propellers, model experiment is unquestionably our surest guide. Here, as in all other cases where we utilise model experiment, we must finally assure ourselves by experience or full-scale experiment that we have a safe law of comparison, but the difficulty of accurate full-scale propeller tests in free flight renders it almost essential for the present that we investigate laws of propeller action by model experiment.

This has been done with good results in the marine field and air propellers are even more favourably circumstanced. For instance, for propellers in water we cannot apply our law of comparison when cavitation is present. Cavitation does not trouble air propellers as yet. The propeller driving a vessel, assuming the atmospheric pressure as equivalent to 34 feet of water, is working in an inelastic fluid under a total head to the centre of propeller of, say, from 35 to 60 feet. The airplane propeller is working in an elastic fluid under a total head,

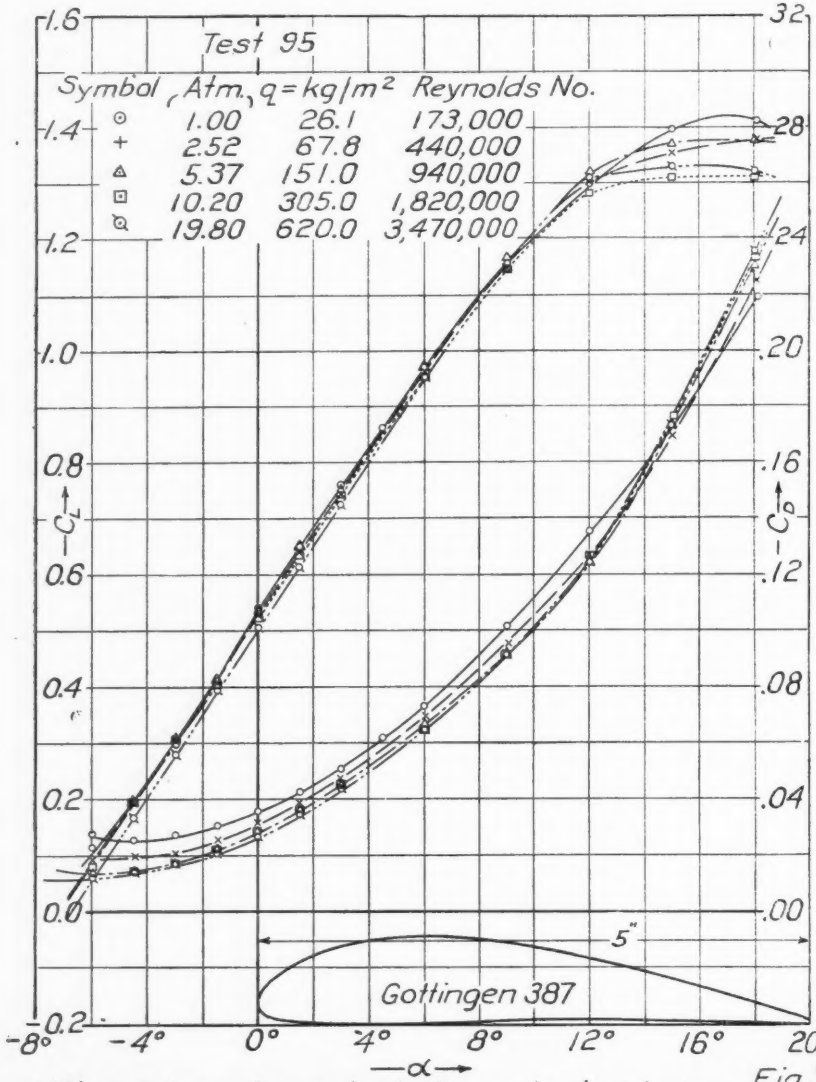


Fig.3 Lift and drag characteristics as tested in the Variable Density Wind Tunnel from 1 to 19.8 atmospheres.

when near the ground, of something like 6 miles. This not only eliminates cavitation but enables us to adopt efficient blade sections that would be impossible in water. The airplane propeller designer is fortunate in this fact and in the further fact that he can use two-bladed propellers. These are capable of more efficiency than three or four-bladed propellers, but can seldom be used for marine propellers because when working in an irregular stream, as they must at the stern of a ship, they are liable to cause excessive vibration of the ship.

Models in water act very much as in air and experiments with thin narrow two-bladed propellers in water show efficiencies fully as good as those of models of airplane propellers. Table II. below gives maximum efficiencies in water of some two-bladed model propellers. They had ogival blade sections, straight faces of uniform pitch and circular arcs for backs, the edges being sharp, not rounded.

TABLE II.

Maximum Efficiencies of Two-Bladed 16in. Model Propellers in Water.

Camber Ratio at .75 Radius.	Mean Width Ratio.	Pitch Ratios. .4	.6	.8	1.0	1.2	1.5
				Maximum Efficiencies.			
.1241	.075	—	—	—	.85	—	—
.0744	.125	—	—	—	.84	—	—
.0465	.200	.56	.71	.78	.82	.83	.84
.0338	.275	—	—	—	.79	—	—

From "Some Results of Tests of Model Propellers," by A. V. Curtis and L. F. Hewins, Transactions Society of Naval Architects and Marine Engineers, Vol. XIII., 1905.

The airplane propeller designer labours under one disadvantage. There is no doubt that as propeller tip speeds in air approach the velocity of sound, we may expect radical departures from the laws of action at lower speeds. That is a complication I shall not attempt to unravel.

Before taking up the model experiment end of propeller action, it may be as well to take up one general consideration.

Since thrust of a propeller is proportional to the sternward momentum per second generated by its action upon the fluid in which it works, it follows that there must be a certain energy carried off in the fluid to which velocity is communicated. Hence, no propeller can show an efficiency of 100 per cent., and the actual efficiency of an ideal propeller at a given speed of advance must always diminish as its thrust increases. Following the treatment proposed by McEntee in 1906 for propellers operating in water, we can gain some idea of limits in air.

Suppose we have an ideal frictionless propelling apparatus which takes hold of the air and discharges it directly aft without change of pressure and with uniform absolute velocity u feet per second, the velocity of advance of our ideal apparatus with reference to undisturbed air being v feet per second. Then if A denotes the area in square feet of the slipstream, the mass of the air acted upon per second is $(w/g)\,A\,(v+u)$.

The thrust T in pounds from Newton's Third Law is equal to the sternward momentum generated per second or $T=(w/g)\,A\,(v+u)\,u$.

Useful work $=Tv=(w/g)\,A\,(v+u)\,uv$.

The lost work or kinetic energy of the air discharged $=(w/g)\,A\,(v+u)\,(u^2/2)$.

Whence gross work $=(w/g)\,A\,(v+u)uv+(w/g)\,A\,(v+u)\,(u^2/2)$.

Efficiency $e=$ useful work/gross work $=v/(v+\frac{1}{2}u)$.

If we solve for u in the expression for thrust we have

$$u=\sqrt{(v^2/4+gT/wA)}-\tfrac{1}{2}v$$

Substituting in the expression for efficiency we have

$$e = 4/\{3 + \sqrt{(4gT/wAv^2 - 1)}\}$$

This then is the general formula for the efficiency of an ideal frictionless propelling apparatus, discharging the fluid passing through it without increase of pressure and accompanying loss of efficiency.

Applying to an air propeller of diameter d feet substitute $\pi d^2/4$ for A. Also give g its standard value of 32.174 ft./sec.^{2}, w the value for standard air of .07651 lbs./ft.^{3}, and express v in miles per hour V instead of feet per second.

When this is done our formula for air becomes

$$e = 4/\{3 + \sqrt{[(16 \times 32.174)/(.07651 \times 2.15\pi) \times T/d^2V^2 + 1]}\}$$
$$= 4/\{3 + \sqrt{(99.56T/d^2V^2 + 1)}\}$$

Fig. 4 shows contours of ideal efficiency derived from the above equations plotted upon values of d and T/V^2.

It is seen that curves of constant efficiency are parabolas, with values of T/V^2 as ordinates and values of diameter as abscissæ; also once we have fixed the diameter and the value of T/V^2 we fix the efficiency. It is obvious also, that if, for a given diameter, we increase thrust without changing speed, or if, for a given diameter, we decrease speed without changing thrust, the efficiency necessarily falls off.

While the above conclusions can be legitimately drawn from Fig. 4, we must not forget that this refers to an ideal propeller of the best possible efficiency. Actual propellers in operation lose not only by the energy carried off in the wake but by their friction and the energy due to transverse motion in the wake, both tending to reduce efficiency. If we assume a law of comparison, which will be discussed later, we can, from tests of a model propeller, draw a diagram similar to Fig. 4, covering the performance of all propellers similar to the model. This is done in Fig. 5 for a propeller of .9 pitch ratio, propeller "E" of Table III., tested by Dr. Durand. It will be observed that in its general features Fig. 5 corresponds fairly well with the ideal diagram of Fig. 4. However, the efficiency contours, instead of increasing indefinitely as we increase the diameter and decrease the values of T/V^2, reach a maximum and then for smaller values of T/V^2 the efficiency falls off very rapidly. Above the maximum line the agreement with the ideal diagram is better. We still have the feature that the efficiency of this family of propellers is dependent upon the diameter and the value of T/V^2. If in level flight we are operating above the parabola of maximum efficiency and undertake to climb, the value of T/V^2 necessarily increases and the efficiency necessarily falls off. If we are operating in level flight in the region below the contour of maximum efficiency and then undertake to climb, the efficiency increases for a time and then falls off as before. In any case, however, the efficiency of a given propeller varies with the flight conditions.

A propeller is an object moving through air and our general equation (4) applies. Rewriting this with T to denote thrust instead of R to denote any resistance, and diameter D in place of L, we have

$$T = \rho V^2 D^2 F_3 (v/DV,\ V^2/V_s^2,\ Dg/V^2,\ r_1,\ r_2, \ldots) \quad . \quad . \quad (4)$$

As before, we can confidently eliminate the gravity variable Dg/V^2. When it comes to the compression variable V^2/V_s^2, we are not upon such sure ground as for the airplane because tip speeds are much greater than airplane speeds. Probably some types of blade section can approach much more closely than others the velocity of sound in air without it being necessary to take account of the compression variable. When that does become necessary the problem can be met by testing propeller models in high speed wind tunnels, V being airplane speed. It would seem from this point of view that even now model propellers

should be tested at full speed. Eliminating the compression variable as not yet important, we come to the same general expression as for the airplane or

$$T = \rho V^2 D^2 F_3 (v/DV)$$

Here we meet the same Reynolds' number complication. Now we have seen in dealing with airfoils that the scale effect correction for them was secondary. We know, too, that in propeller action, where a blade is attacking air already somewhat disturbed by its previous passage or the passage of another blade, we may expect great turbulence in the action and we know that the greater the turbulence the less the scale effect correction for viscosity.

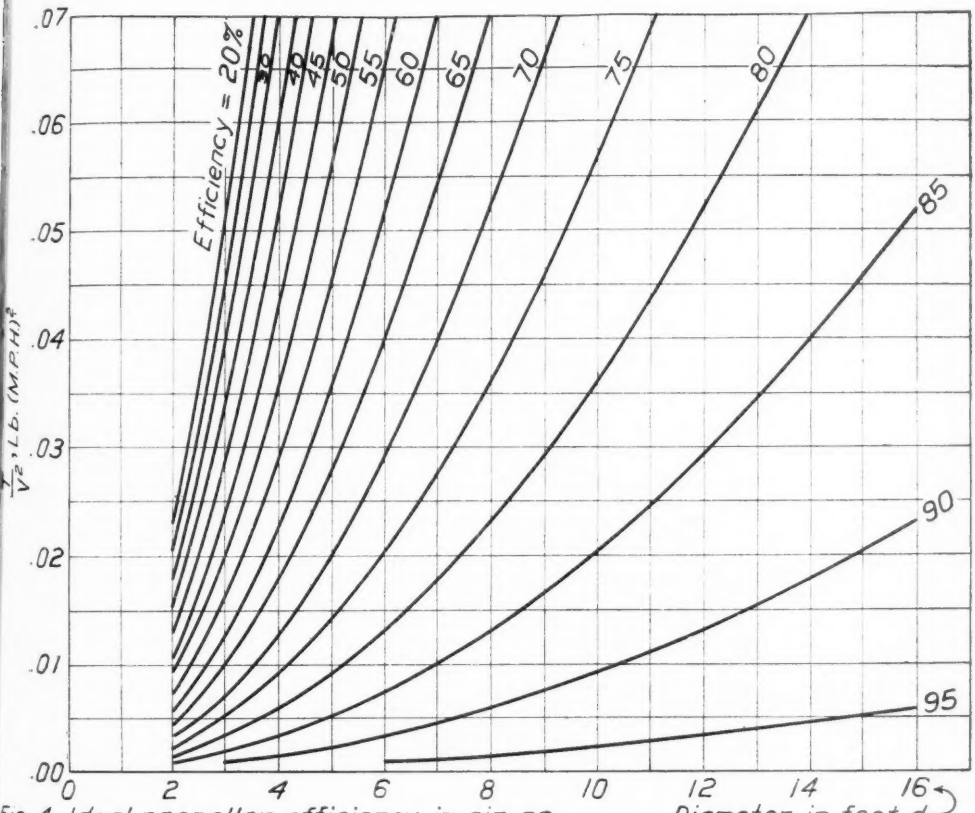


Fig. 4 Ideal propeller efficiency in air as affected by thrust in pounds, T speed in M.P.H, V and diameter in feet, d

So we conclude that if we drop the Reynolds' number variable we have left a first approximation sufficiently close to exactness for practical present day engineering purposes.

This leads us to the simple expression

$$T = \rho V^2 D^2 F_3 (r_1, r_2, \ldots)$$

If this were exact, a model of any size at any speed would tell us all we need know, but bearing in mind that our expression is approximate and the nature of

the quantities ignored, we should make our models as large as possible and test them at as high a speed as possible.

The quantities r_1, r_2, etc., are characteristic ratios and it is very important if we are to utilise model propeller tests to best advantage that we use ratios that are truly and adequately characteristic. Systematic treatment is necessary here. Much progress has been made in connection with aeronautic propellers by regarding their blades as composed of sections of airfoils, and this is essentially a fruitful method of procedure in our search for more efficient forms. It is interesting to note that the underlying idea is the same as that of Mr. William

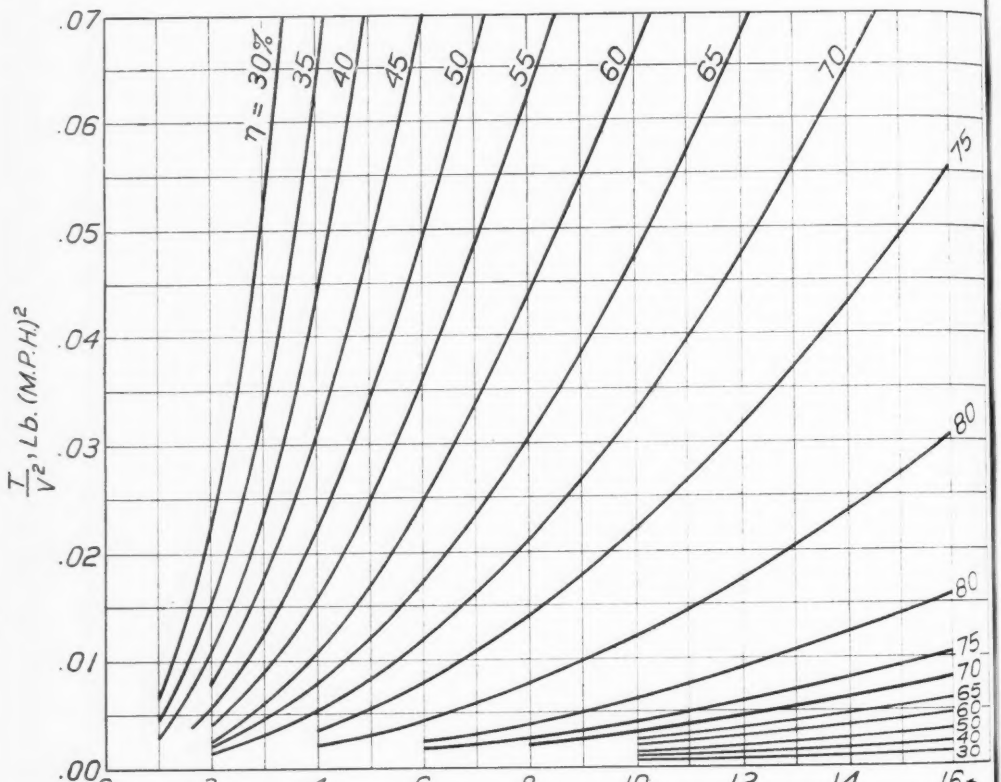


Fig. 5 Actual propeller efficiency in air derived from model propeller tests - propeller E table III

Froude, when for the marine propeller, in 1878, he put forward his blade theory, in which the propeller blades were regarded as made up of plane elements advancing through the water. Mr. Froude at that time said, with much justice:—

> "No theoretical treatment of the action of an actual screw can be sound which does not incorporate and mainly rest on the principles embodied in the treatment of the problem of the plane, and indeed the character of the results must, in their most essential features, be the same in both cases."

The fundamental difficulty with the Froude blade theory was that propeller blades are not a plane of no thickness. This fact has been fully recognised by aeronautic designers in treating them as made up of airfoil elements, but after all the model propeller and the full-sized propeller must each be treated in the end as a whole, and our ratios must be based upon that fact.

It is obviously desirable to have diameter appear in all ratios if possible, since diameter is our basic dimension.

The first thing we need to consider is the most desirable ratio by which to express the obliquities of the blades, or the angles which they make with the axis of the propeller. This is sometimes done by stating the angles, but the best plan appears to be to adhere to the idea of pitch. This tends to become absurd in propellers with blades relatively as thick as those used on airplanes, but after all it is always possible at any section to establish a line making a definite angle with the axis. If the propeller has a working face with any material portion of it flat or straight in section on the driving face, this is naturally the line used to express pitch. However that may be, for the family of propellers derived from a given model, the ratio between pitch and diameter is always constant, and as a rule the whole family may be characterised by the extreme pitch ratio. This is about the simplest quantity we can use which gives an idea of the general features of the propeller with reference to the pitch, or blade obliquity if we prefer that expression. Then one of our ratios is the ratio between pitch and diameter, denoted by a.

We need something expressing relative blade width. Aspect ratio will do it, but there seems no necessity for departing from what is frequently called in marine propellers the mean width ratio, namely, the ratio between the mean or average width of the blade and the diameter. If we were always dealing with blades of the same developed outline, it would be simpler and better to use the ratio between the maximum width, a thing we need always to know and use, and the diameter. This would quite well characterise the propeller, but does not seem to be quite so good for universal use in view of variations in blade outline.

We come now to the most difficult ratio to express in practice. We need something to characterise the blade thickness. If propellers all had radially straight faces and straight backs, the simplest and obvious plan would be to extend the line of the face to the axis, the line of the back at maximum thickness to the axis, and express the characteristics of the propeller as regards blade thickness by the ratio between the intercept on the axis thus obtained and the diameter. This is a method which has come into a good deal of use for marine propellers of late years, but the backs of aeronautical propellers vary so much that it is doubtful if we are yet ready to adopt this as a standard ratio. I suggest tentatively for this last ratio the camber ratio at $\frac{3}{4}$ of the radius.

It is now necessary to consider what to do with results of model tests.

These results, such as curves of thrust and torque or dimensionless coefficients derived from them, including curves of efficiency of propeller, are usually plotted initially upon the dimensionless quantity V/nD when V is speed of advance of the propeller with reference to undisturbed air in feet per second, n denotes revolutions per second, and D is diameter of the propeller in feet. Now V/nD is a natural coefficient and excellent as a basic variable when we are dealing with one propeller, but when dealing with systematic propeller research and making diagrams for design purposes it is somewhat lacking. For a single propeller, when we plot upon V/nD we are virtually plotting upon the slip ratio s, since if a denote pitch ratio $V/nD = a\,(1-s)$.

The question of the basic variable to be used in plotting experimental data for design purposes is a very important one and worthy of a little examination. In the first place this basic variable must be dimensionless since we wish to use

model results for dealing with full-sized propellers. There are any number of dimensionless functions available and they are readily converted one to another.

In the second place, looking at the matter from the design point of view, our basic variable should take account of or involve *all* the quantities known or assumed upon which a propeller design depends. Here we meet the fact that we do not necessarily base a propeller design always upon the same quantities. However, considering airplane, propeller, and motor separately, let us see what quantities we have.

For the airplane we have speed, drag or resistance, and effective or useful power used in overcoming the drag at the speed of the airplane. For the propeller we have speed, revolutions, torque, thrust, power absorbed by torque and power delivered by thrust. For the motor we have revolutions, torque and power delivered to and absorbed by the propeller.

Now the propeller is the middleman as it were and may be considered as driving the airplane or as absorbing the motor power. From the first point of view we need a basic variable involving speed, revolutions and either thrust or power delivered by thrust. From the second point of view our basic variable should involve speed, revolutions and either torque or power absorbed by the propeller.

It appears then that to meet all contingencies we really need two basic variables and obviously they should be readily convertible or connected by a simple relation. This indicates that our two basic variables should both involve speed and revolutions and then one should involve torque and the other thrust, or one should involve power absorbed by the propeller and the other power delivered by the propeller or useful power. Each set has its advantages, but the set involving power seems preferable for two reasons. When we deal with motors we normally deal with power, not torque, and the relation between power delivered to and delivered by the propeller is very simple, being the efficiency of the propeller with no intervening factor.

Having settled upon the quantities to enter into our basic variable its form is readily determined by applying the principle of dimensional homogeneity. Consider the variable $P^X R^Y V^Z \rho$ where P denotes power; R, revolutions; and V, speed; and the exponents X, Y and Z are to be determined. The dimensions of P are $ml^2 t^{-3}$; of R, t^{-1}; and of V, $l t^{-1}$. Then dimensionally $P^X R^Y V^Z = m^X l^{2X} t^{-3X} t^{-Y} l^Z t^{-Z} m^{-l3}$.

$$X+1=0 \qquad 2X+Z-3=0 \qquad -3X-Y-Z=0$$
$$X=-1 \qquad Y=-2 \qquad Z=5$$

Our expression is $\rho V^5/PR^2$. This or the equivalent is well known and has been used more or less for many years. Of course we can use the reciprocal or any power. For marine propellers a very convenient expression is $1/\sqrt{(\rho V^5/PR^2)}$, or calling $\rho=1$, $(R\sqrt{P})/V^{2\frac{1}{2}}$.

For aeronautic work we need to keep ρ and from a practical point of view there seem to be some advantages, as in marine work, in using expressions where R appears in the numerator and in the first power.

These considerations lead us to the expressions below for basic variables.

Based on motor power $(R\sqrt{P})/\sqrt{(\rho V^5)}$.
Based on useful power $(R\sqrt{U})/\sqrt{(\rho V^5)}$.

Now we must select the essential things to be plotted upon our basic variable. Efficiency is one, of course. The other quantity that we need is some dimensionless function involving diameter—our primary dimension—and preferably it should involve diameter in the first power and in the numerator.

Such a function is DR/V, which we may call δ. When we come to plot efficiency and δ upon our basic variable we find it desirable to use logarithmic scales to keep curves within manageable limits.

The most fruitful model propeller experiments are those made with series of propellers changing one variable at a time. The field is too vast to cover it fully in this way, but experiment soon shows that some of our variables are primary in their effects, while others are rather secondary, and may be mainly taken account of in a first approximation without special experiments. Experiments made with a group of propeller models varying systematically were recently completed by Drs. Durand and Lesley for the U.S. National Advisory Committee for Aeronautics, and illustrate the points referred to above. Thirteen propellers, "A" to "M," were tested. They are shown by projection and sections in Figs. 6 to 9 inclusive, and Table III. below gives their essential characteristics.

TABLE III.

Model Propellers Tested by Durand and Lesley.

Designation.	Pitch Ratio.	Maximum Width Ratio.	Mean Width Ratio.	Camber Ratio at .75 Radius.
A	.5	.0833	.070	.107
B	.6	.0833	.070	.107
C	.7	.0833	.070	.107
D	.8	.0833	.070	.107
E	.9	.0833	.070	.107
F	1.0	.0833	.070	.107
G	1.1	.0833	.070	.107
H	.7	.0833	.070	.118
I	.7	.0833	.070	.128
J	.7	.0833	.070	.139
K	.7	.0666	.046	.107
L	.7	.1000	.085	.107
M	.7	.0769	.066	.107

It will be observed that propellers "A" to "G" inclusive have essentially the same blade sections similarly distributed radially, but differ in pitch, the pitch ratios running from .5 to 1.1. This makes seven propellers with variation in pitch as the primary characteristic. Propellers "H" to "M" inclusive, and also "C," all have the same pitch ratio, the differences being in mean width ratio and thickness, expressed by camber ratio at .75 radius.

Figs. 10 and 11 show the results for the propellers of varying pitch plotted as non-dimensional coefficients upon the basic variables deduced above. Fig. 12 shows the seven propellers of uniform pitch but varying blade sections plotted in the same manner upon the basic characteristic for power only. We see from Figs. 10 and 11 that the possible efficiency of an airplane propeller is essentially a question not of propeller design but of the requirements to be met by the propeller. Given the power to be absorbed or delivered, the speed, and the revolutions per minute for this family of propellers, and the maximum efficiency attainable is fixed, and it may well happen that it will fall below the 80 per cent. efficiency, which is sometimes regarded as normal. Of course Figs. 10 and 11 refer to only one family of propellers, but it will be found that almost any family will plot in the same general way. The efficiencies may be a little higher or a little lower, but the variations of efficiency will follow closely the variations of Figs. 10 and 11.

Studying these figures, it will be found that for a given combination of power, speed and revolutions a definite pitch ratio shows the maximum efficiency, but there is a relatively wide range of pitch ratio on each side of that for maximum efficiency where the falling off is slight. Keeping revolutions, power and speed the same, we may use a smaller propeller of coarser pitch or a larger propeller of

finer pitch without a reduction in efficiency of more than a point or so, an amount which could hardly be detected in service.

In view of Figs. 10 and 11, inspection of the propellers of the original Wright plane produces admiration of the engineering genius of the pioneers of the air. This low speed plane has two relatively very large propellers of coarse pitch. These characteristics are essential to the best efficiency under the condi-

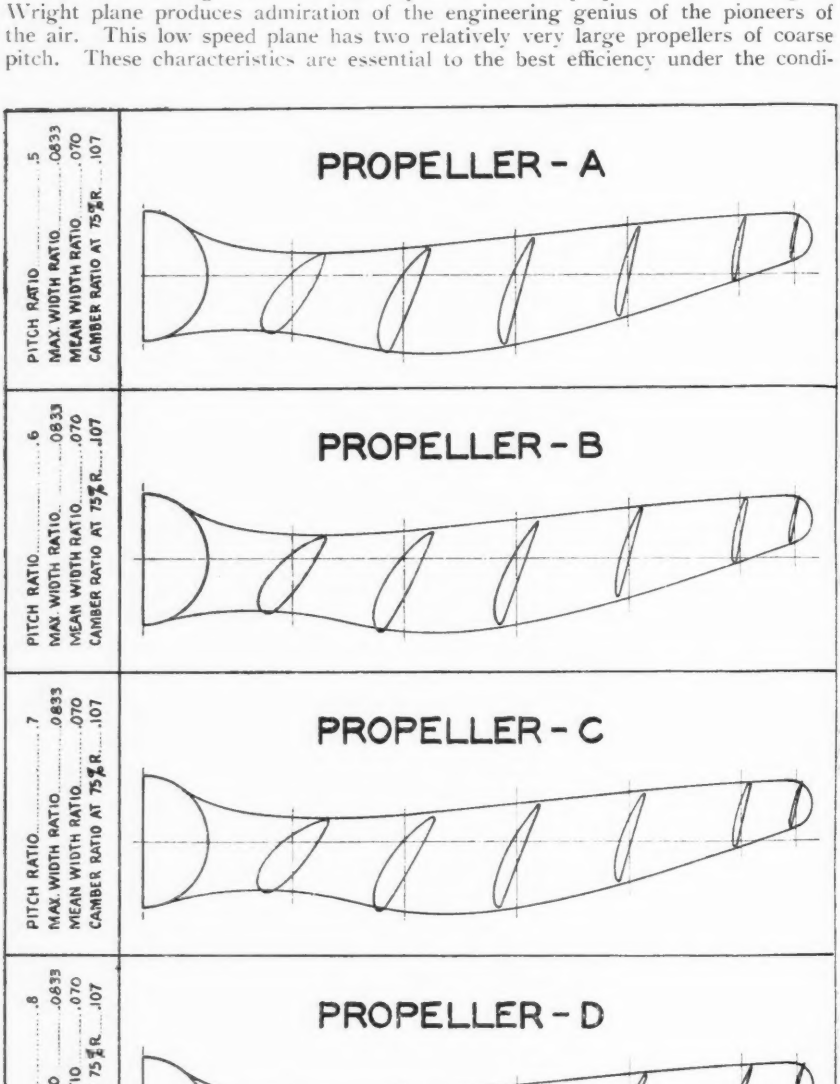


Fig. 6

tions to be met. Fast planes of the present day may obtain good efficiency with single propellers of high revolutions and fine pitch, but if the Wrights had fitted such a propeller their plane probably would not have flown at all.

Coming now to Fig. 12, it will be observed that the variation of efficiency is remarkably small for the variations of blade section of all seven propellers, "C" and "H" to "M" inclusive. The differences are almost within the limits

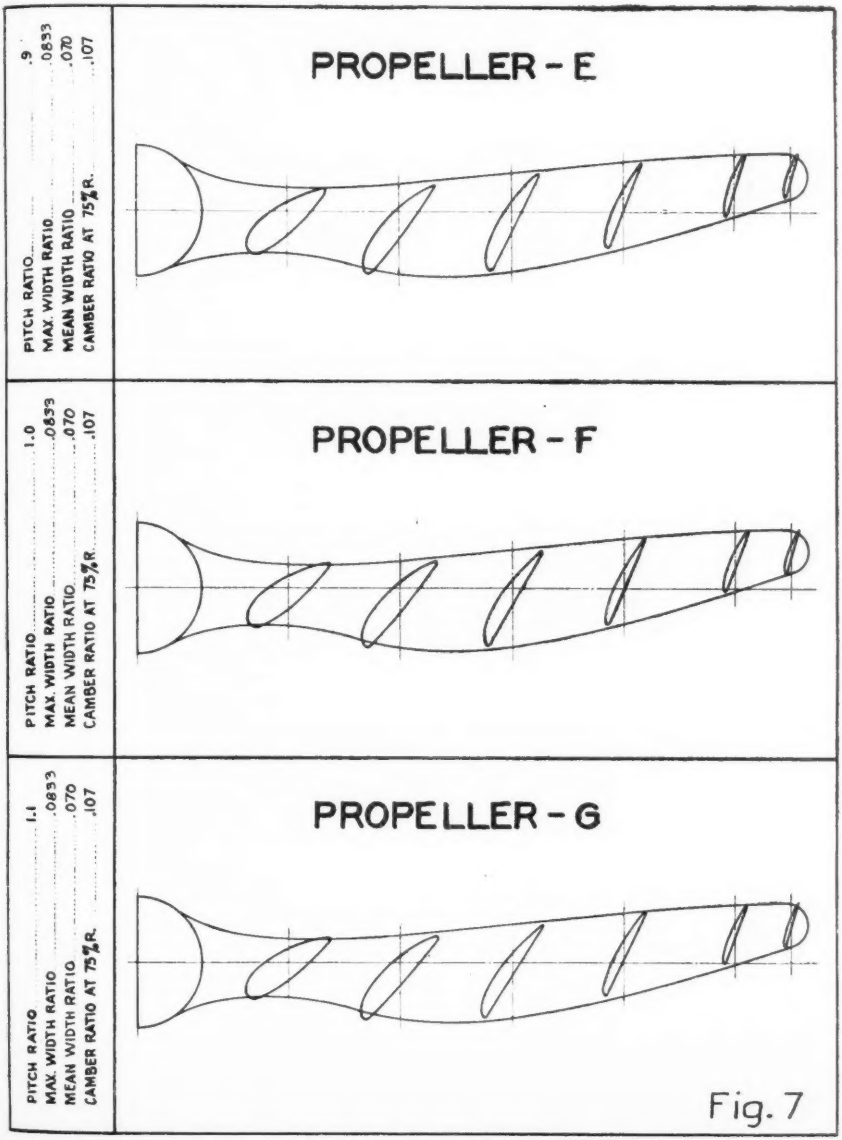

PROPELLER - E
PITCH RATIO 9
MAX. WIDTH RATIO ..... .0833
MEAN WIDTH RATIO ..... .070
CAMBER RATIO AT 75% R. ..... .107
PROPELLER - F
PITCH RATIO 1.0
MAX. WIDTH RATIO ..... .0833
MEAN WIDTH RATIO ..... .070
CAMBER RATIO AT 75% R. ..... .107
PROPELLER - G
PITCH RATIO 1.1
MAX. WIDTH RATIO ..... .0833
MEAN WIDTH RATIO ..... .070
CAMBER RATIO AT 75% R. ..... .107

Fig. 7

of error to be expected in such experiments. As regards diameter, the variation resulting from change of section is normal, the thicker blades requiring smaller diameter because their virtual pitch ratio is greater. Thin blades and narrow blades, similarly, act as normal blades of slightly greater diameter. The efficiency curves in all three figures show minor inconsistencies which could readily be faired out. They were taken from curves plotted on entirely different variables.

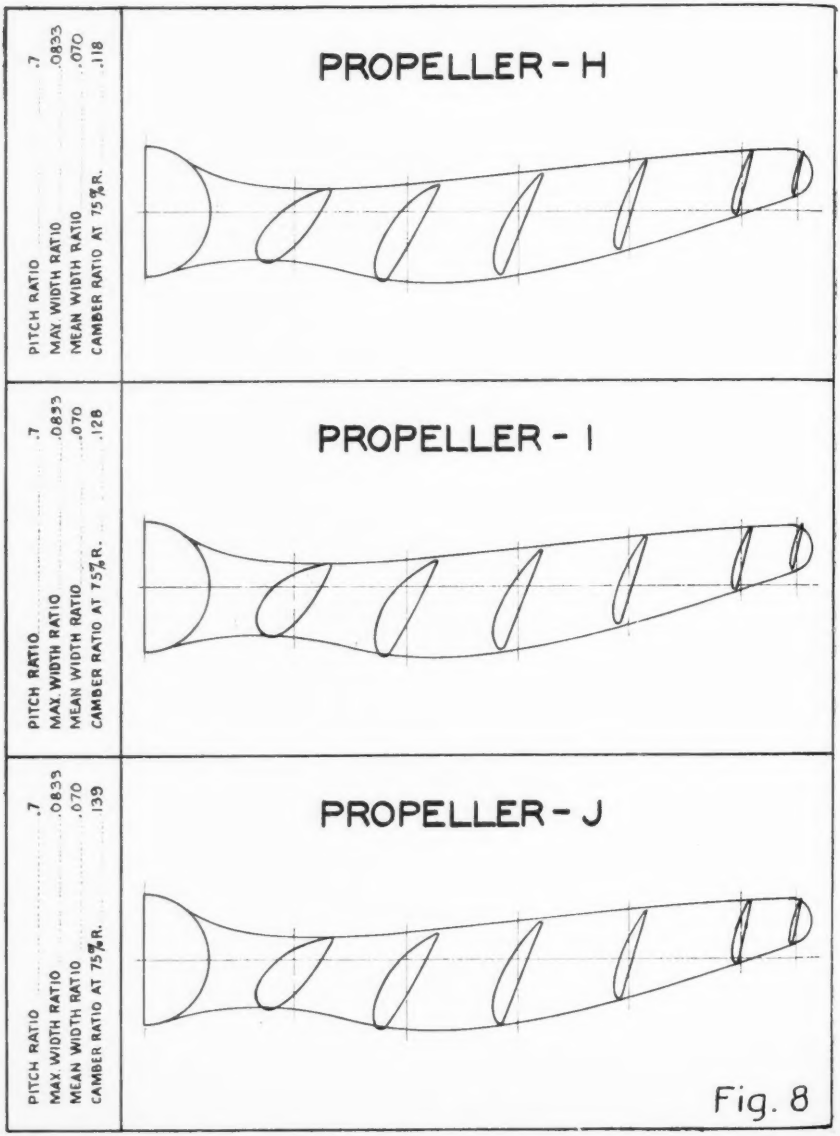


Fig. 8

In concluding this part of my subject, it might be pointed out that systematic diagrams such as Figs. 10 and 11, from one family of propellers, may be used to extrapolate with a good deal of accuracy the results to be expected from propellers of another blade type when but one of the type has been tested. If, for instance, the one tested has a pitch ratio of .7, we will say, and its δ line falls 3 per cent. above or below the δ line for the .7 pitch ratio in Figs. 10 and 11 we may conclude

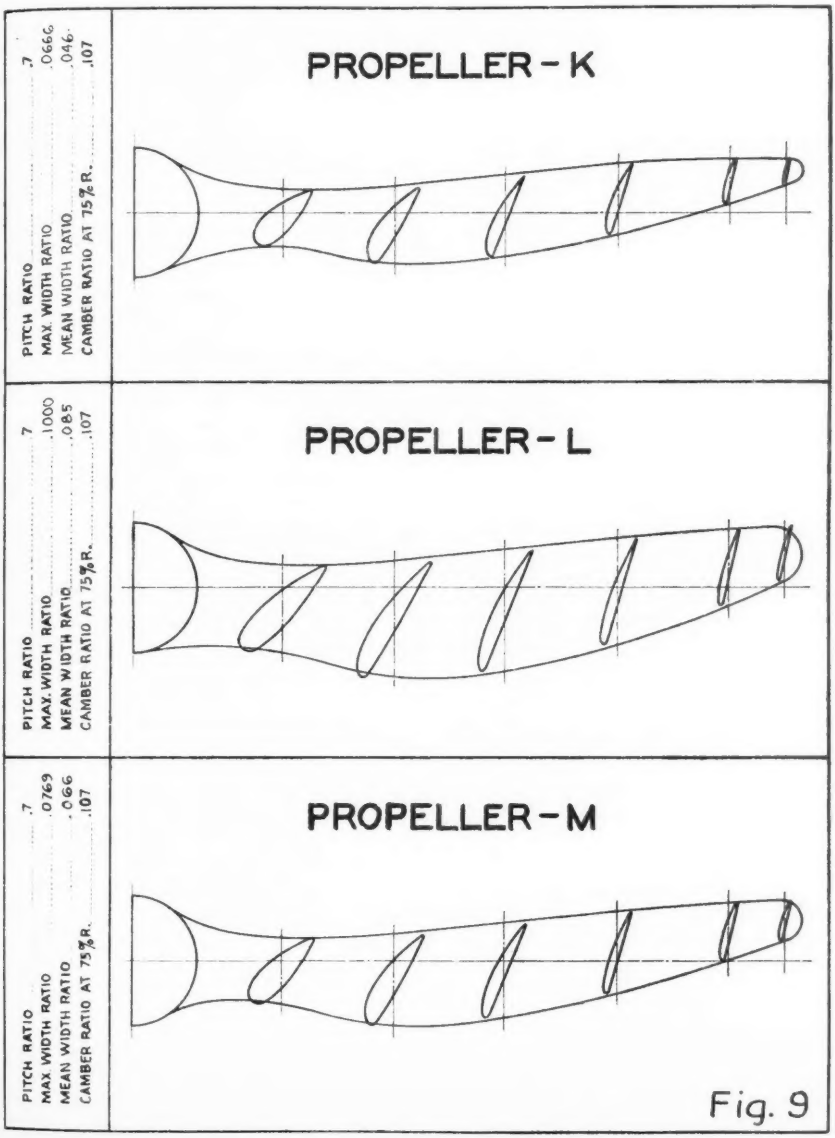
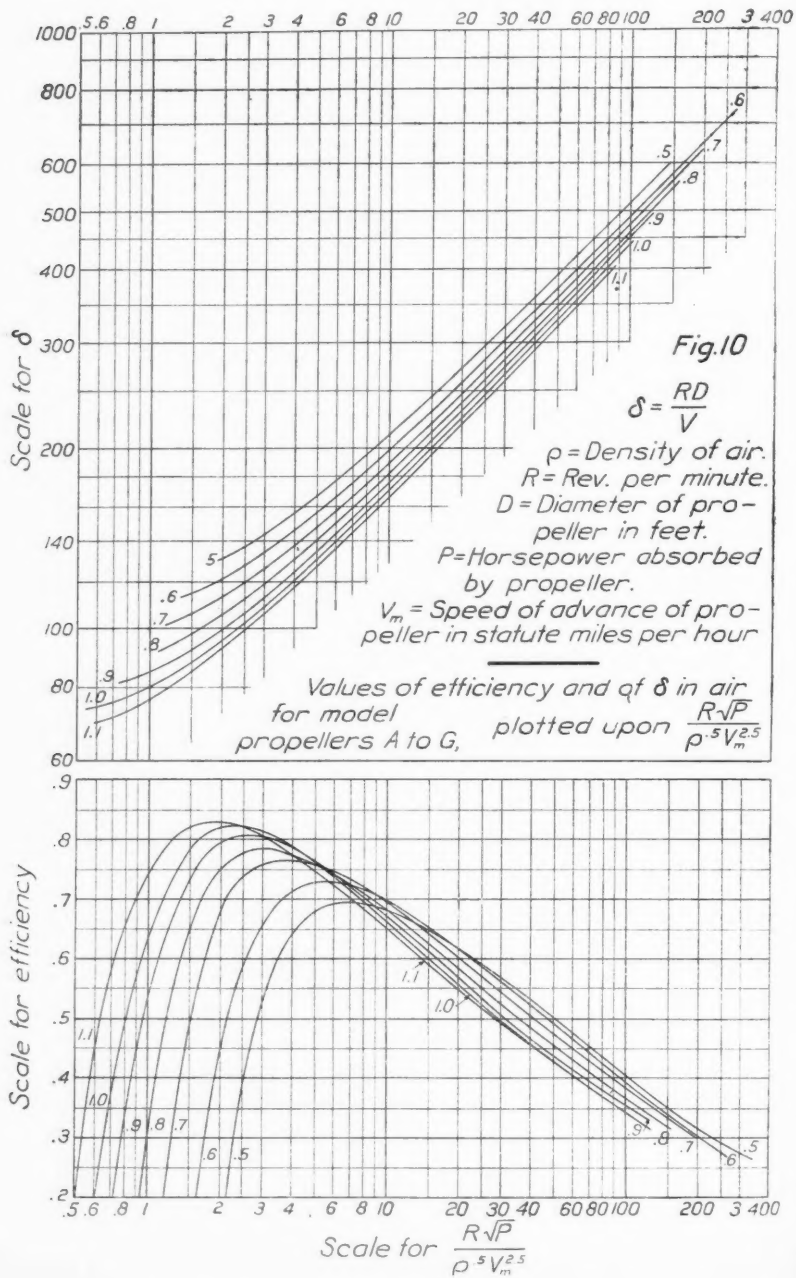

PROPELLER - K
PITCH RATIO .7
MAX. WIDTH RATIO .0666
MEAN WIDTH RATIO .046
CAMBER RATIO AT 75% R. .107
PROPELLER - L
PITCH RATIO .7
MAX. WIDTH RATIO .1000
MEAN WIDTH RATIO .085
CAMBER RATIO AT 75% R. .107
PROPELLER - M
PITCH RATIO .7
MAX. WIDTH RATIO .0769
MEAN WIDTH RATIO .066
CAMBER RATIO AT 75% R. .107

Fig. 9


Scale for δ
Scale for efficiency
Scale for $\frac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$
Fig.10
$\delta = \frac{RD}{V}$
ρ = Density of air.
R = Rev. per minute.
D = Diameter of propeller in feet.
P = Horsepower absorbed by propeller.
V_m = Speed of advance of propeller in statute miles per hour
Values of efficiency and of δ in air for model propellers A to G, plotted upon $\frac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$

with good approximation that the same relation will hold for pitch ratios of .6 and .8. It is rather remarkable at first sight to see how δ lines for propellers of quite different blade sections tend to parallel one another when plotted upon the basic variables used and how little efficiency is affected by variations of blade sections, etc.

But after all when we go back to first principles these results are perfectly natural.

In air the pressure in the slipstream cannot differ much from the undisturbed pressure of the air. Then the thrust is proportional to the sternward momentum, and as shown in Fig. 4, there is a certain unavoidable loss or waste of power associated with it even if we had a perfect propelling instrument. With actual propellers, that are not perfect, we have two further losses due to edge or frictional resistance of the blades and to the transverse momentum communicated to the air involving energy. Neither of these two further losses can be eliminated and from the nature of the case it does not seem that there is a large field for reducing them, though there is plenty of opportunity to increase them.

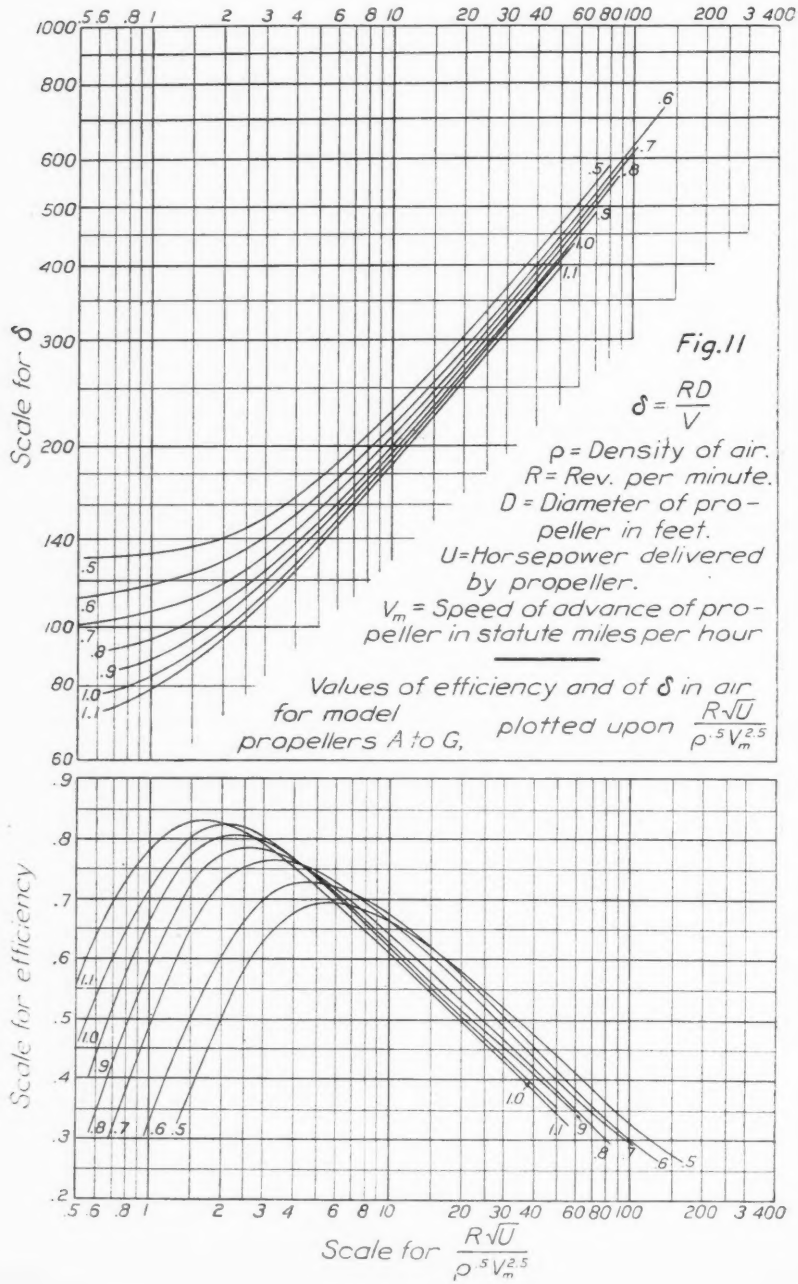
As tip speeds increase it will be more and more important to develop types of blade section to avoid the quasi cavitation that must be guarded against.

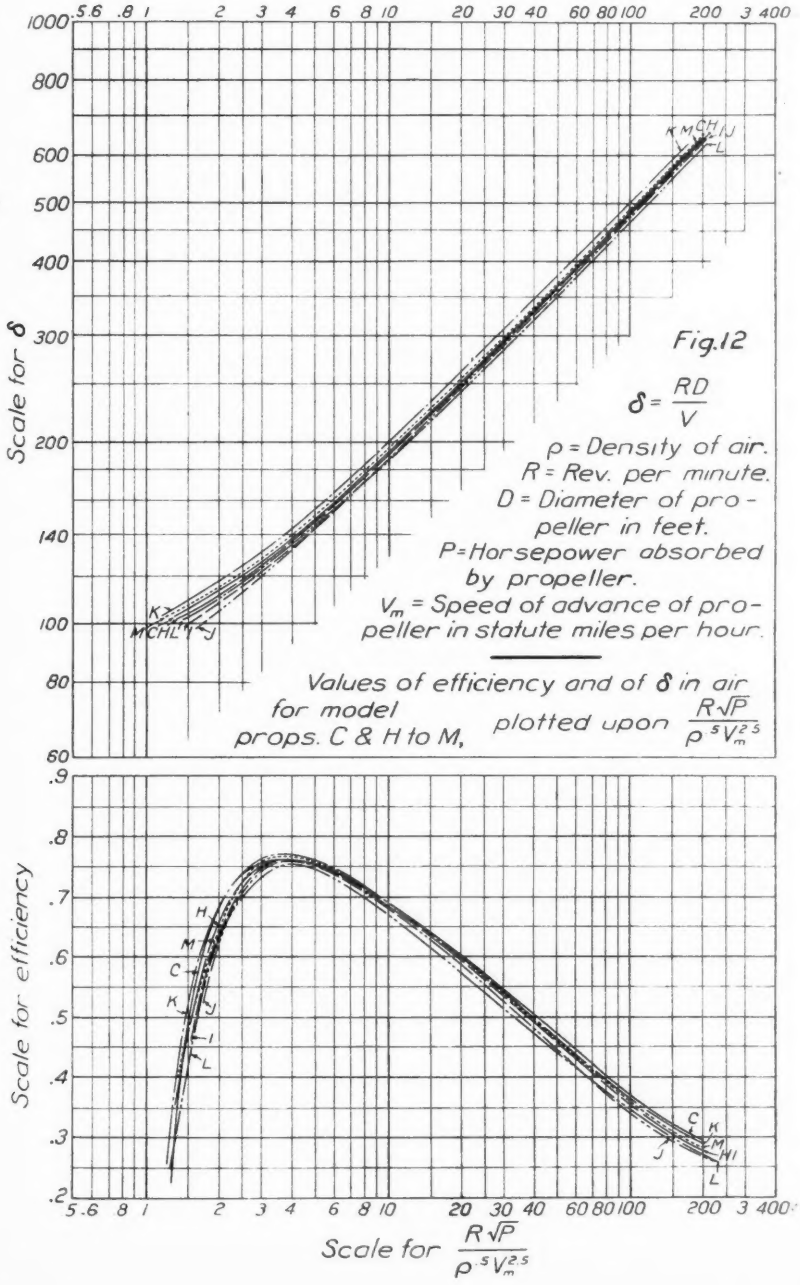
It may be recalled that several times reference has been made to the difficulties of satisfactory full-scale trials. However, we can never rely absolutely upon model experiments until they have been checked by corresponding full-scale trials. During the last year the National Advisory Committee for Aeronautics has attempted such a comparison, the model experiments and the full-scale tests being both carried out by Professor E. P. Lesley. The full-scale experiments, with five airplane propellers on a VE.7 airplane with a Wright E.4 engine, were conducted at the Langley Memorial Aeronautical Laboratory between May 1st and August 30th, 1924. The model experiments were carried out subsequently with models of the same propellers and also a partial model of that part of the airplane exposed to the slipstream, the model being on the scale of 0.3674.

Heretofore we have considered propeller performance alone. When we come to combine the propeller and its airplane, we meet the complication that each reacts upon and affects the performance of the other. The slipstream from the propeller affects its airplane for tractor propellers, increasing the resistance and somewhat disturbing the balance. This can usually be expressed as regards propulsion matters as an augment of the drag. Furthermore, the disturbance set up in the air by the airplane extends to the air around the propeller, the net result being that the propeller, instead of moving uniformly through the air at the speed of the airplane, moves through air variously disturbed. For the tractor propeller the net result is that the air acted upon by the propeller has already had its relative velocity more or less checked by the reaction from the airplane.

About the only practical way to deal with this matter is to regard this disturbance as equivalent to a uniform slowing up of the air, so that a propeller, instead of behaving as if it were passing through still air with velocity V of the airplane, behaves as if it were passing through still air with a velocity V_1, less than that of the airplane.

When we come to consider the efficiency of the combination, it is unfortunately necessary to make a clear distinction between the efficiency of the propeller and the "efficiency of propulsion." The efficiency of propulsion is best regarded as the ratio between the power delivered to the propeller and the power necessary to propel the airplane under the circumstances if there were no propeller acting. The efficiency of the propeller, however, is the ratio between the useful power which it delivers and the power delivered to it. The power which it delivers depends upon its actual thrust and its speed, V_1, through the air upon which it acts. The thrust is normally greater than the drag of the airplane without the propeller, and V_1 is normally less than V. These two factors affect


Scale for δ
Scale for efficiency
Scale for $\frac{R\sqrt{U}}{\rho^{.5}V_m^{2.5}}$
Fig.11
$\delta = \frac{RD}{V}$
ρ = Density of air.
R = Rev. per minute.
D = Diameter of propeller in feet.
U = Horsepower delivered by propeller.
V_m = Speed of advance of propeller in statute miles per hour
Values of efficiency and of δ in air for model propellers A to G, plotted upon $\frac{R\sqrt{U}}{\rho^{.5}V_m^{2.5}}$


Scale for δ
Scale for efficiency
Scale for $\frac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$
Fig.12
$\delta = \frac{RD}{V}$
ρ = Density of air.
R = Rev. per minute.
D = Diameter of propeller in feet.
P = Horsepower absorbed by propeller.
V_m = Speed of advance of propeller in statute miles per hour.
Values of efficiency and of δ in air for model props. C & H to M, plotted upon $\frac{R\sqrt{P}}{\rho^{.5}V_m^{2.5}}$

efficiency in opposite directions, and the result is that the efficiency of propulsion may be greater or less than that of the propeller, according to circumstances. In practice we may usually expect to find it somewhat less.

In the free flight experiments at Langley Field it was necessary first to determine the drag of the airplane. This was done over a range of speed from

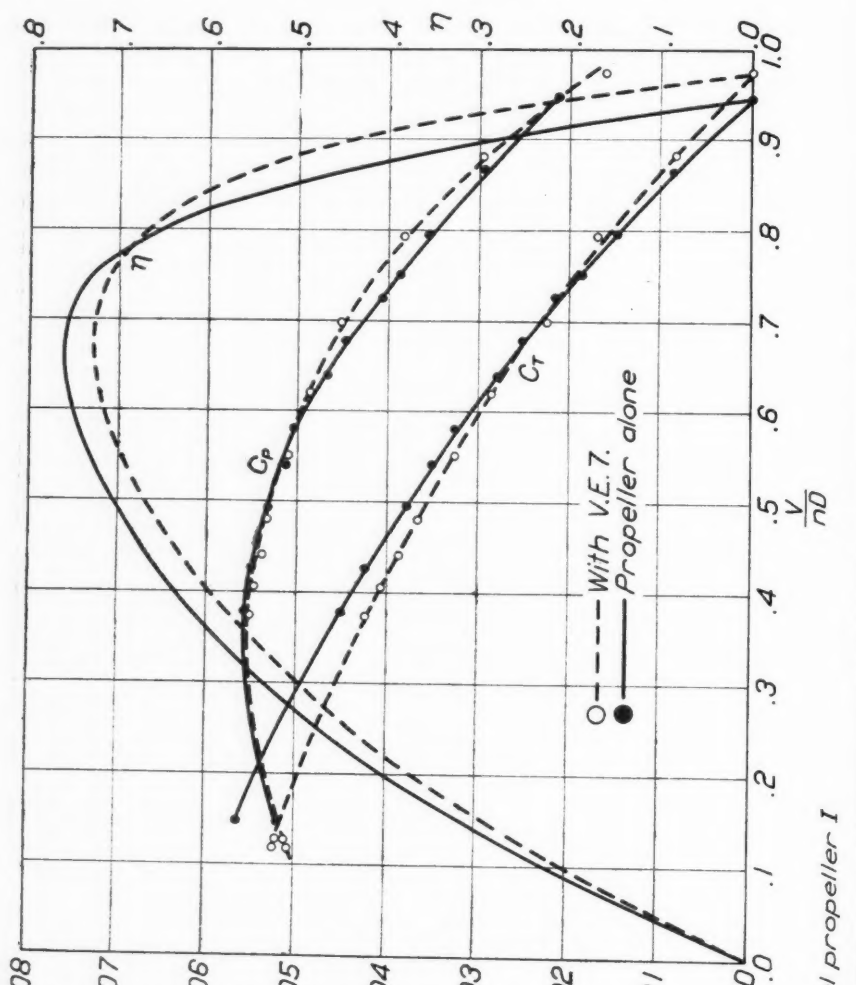


Fig. 13 Model propeller I

50 to 135 miles per hour by making steady glides at various steady angles, the propeller being throttled until the thrust was very close to zero, correction being subsequently made for its departure from zero. This being done, it was possible, from the angle of glide and velocity through the air during the glide, both of which were carefully measured, to determine the drag and the ratio between lift

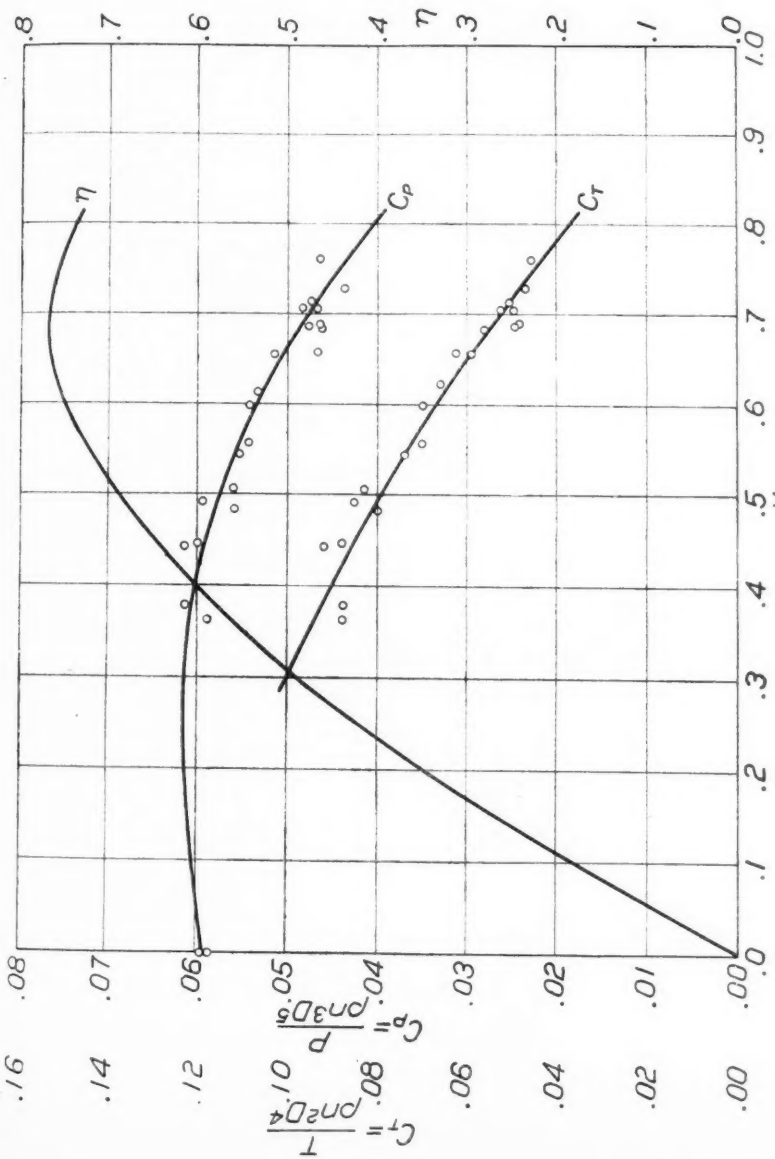


Fig. 14 Propeller I full scale with V.E.7.

and drag. The power could be determined only indirectly by means of careful calibration runs of the airplane engine on the testing stand with full throttle. The power flights which were used for reduction were made at full throttle, consisting of runs at air speeds from 50 to 135 miles per hour in level flight, climb, or power dive, as determined by the speed. It being impracticable in the

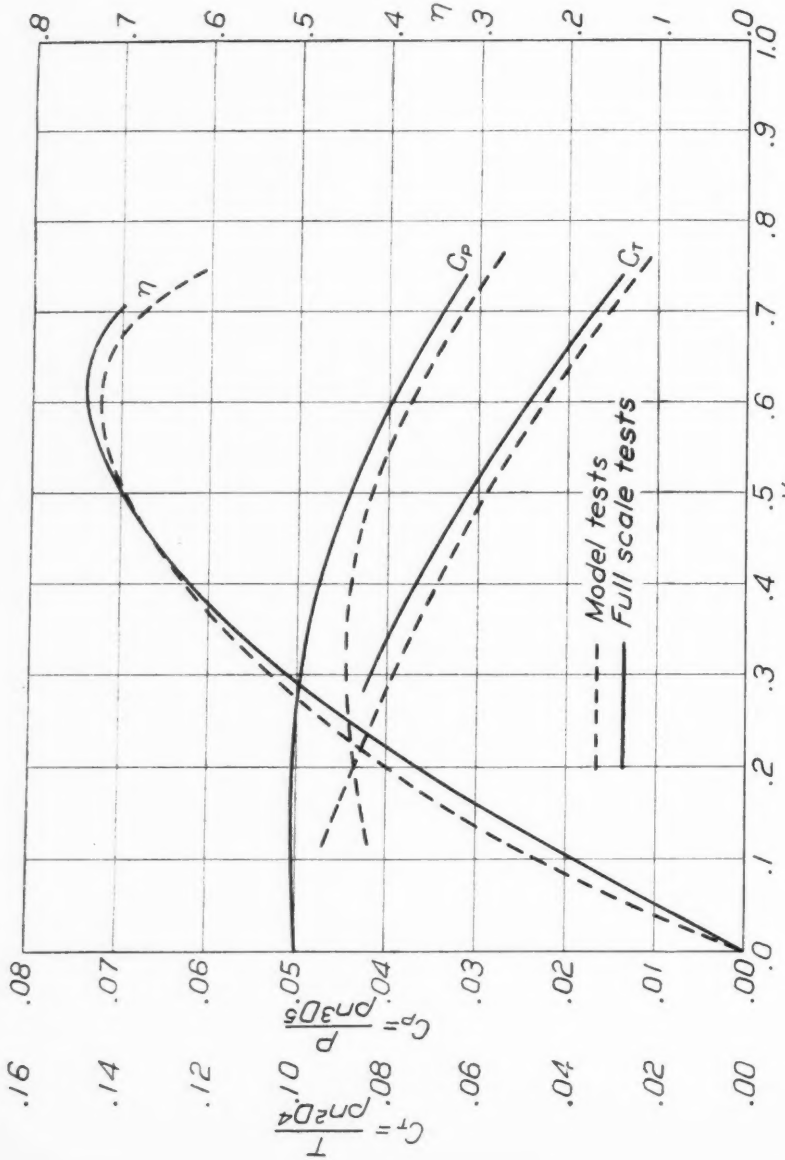


Fig. 15 Propeller B'

full-scale trials to determine the effect of the slipstream upon the airplane or the airplane upon the propeller, the efficiency in the air was regarded as the efficiency of propulsion, not the efficiency of the propeller.

The model tests were made with models of the same five propellers used in the air and, in order to make them comparable with the free flight tests, they

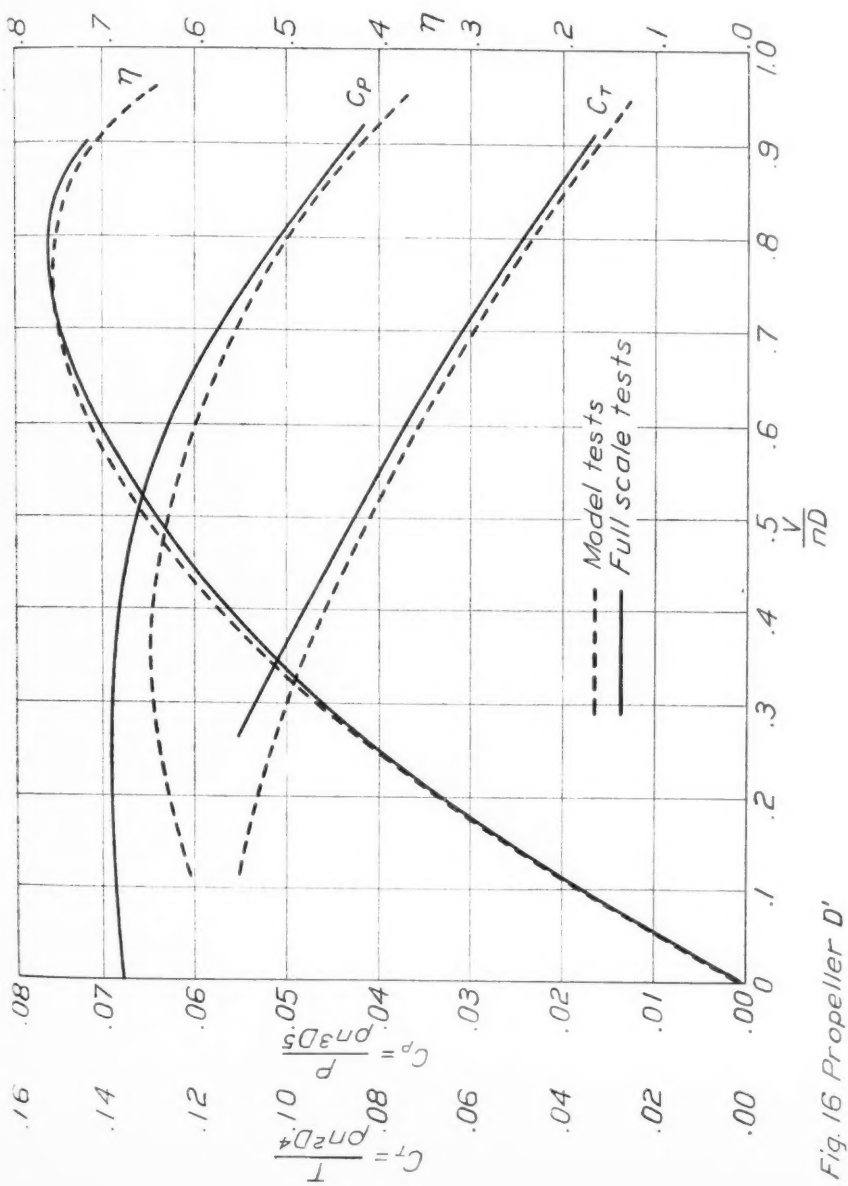


Fig. 16 Propeller D'

Fig. 15 Propeller B'

also were reduced to an efficiency of propulsion. That is to say, a thrust coefficient was obtained by using the net thrust, which was the actual thrust less the difference between the drag of the airplane model with the propeller working and its drag without the propeller, all being plotted upon the basis of the speed through the air. The five propellers used had the dimensions and coefficients given in the table below.

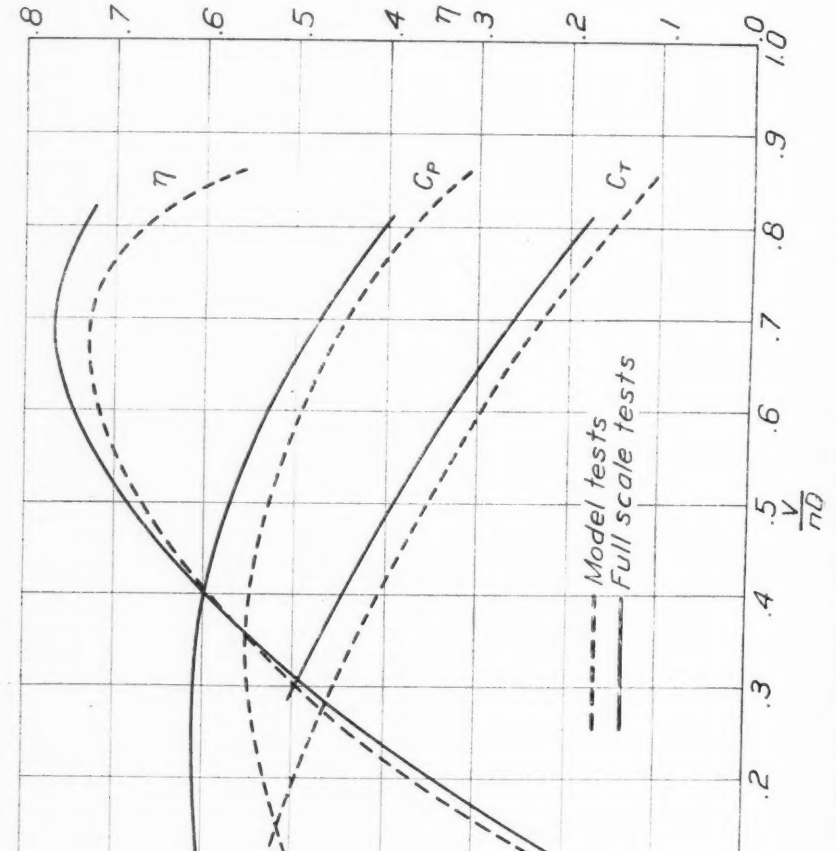


Fig. 17 Propeller I

TABLE IV.

Data of Five Propellers Used in Free Flight Tests.

Designation.	Diameter.	Pitch.	Pitch Ratio.	Maximum Width Ratio.	Mean Width Ratio.	Camber Ratio at .75 Radius.
B′	8′ 6″	5′ 1½″	.6	.0833	.070	.128
D′	7′ 10″	6′ 3.2″	.8	.0833	.070	.128
I	8′ 2″	5′ 8.6″	.7	.0833	.070	.128
K′	8′ 2″	5′ 8.6″	.7	.0666	.046	.128
L′	8′ 2″	5′ 8.6″	.7	.1000	.085	.128

The model results generally were quite consistent.

Fig. 13 shows the results of model propeller "I" with the partial VE.7 model in place, compared with the model results of the propeller alone. In Figs. 13 to 19 inclusive, P, upon which values of C_P depend, does not mean power, but foot pounds per second, or 550 × power. T denotes thrust in pounds. Of course as regards the propeller, abscissæ of V/nD mean something slightly different according as the airplane model is or is not present. With the propeller alone it refers to the true slip; with the propeller and model it refers to the apparent slip, and the case is also affected by the use of the net thrust instead of the actual thrust. In the full-scale work we cannot determine the actual drag and the air speed relative to the propeller.

Fig. 14 shows for propeller "I" in free flight the curves of C_T, C_P, and corresponding efficiency of propulsion on the basis already explained. Finally, in Figs. 15 to 19 there are brought together the results of the model tests with model of plane in place, and the free flight tests in the shape of curves of C_T, C_P, and efficiency. It will be observed that in each case the coefficients are larger in free flight than as estimated from the model results. While the differences vary, as is to be expected, they are consistently too great to be accidental, averaging somewhat on the order of eight per cent., although for propeller D′ they are very small. It is significant that the efficiency differences are very small indeed. Without entering too much into the realm of speculation, it may be pointed out that there are several more or less constant perturbing causes. One is the scale effect; another is the fact that in the model propeller tests the propeller shaft is always parallel to the direction of the flight, whereas in the flight tests the angle made by the propeller shaft with the flight path varied between zero and ten degrees. Inspection of Figs. 15 to 19, however, indicates that the perturbation, broadly speaking, increases with the thrust; that is to say, it increases as V/nD decreases. This points to a third cause of perturbation, namely, the elastic deformation of the blades of the propeller under stress, a deformation that would be much greater on the full-sized propeller than on the model tested at less than full speed. A moderate deformation of the full-sized propellers would account for all the discrepancies in Figs. 15 to 19.

There are, however, too many uncertainties in such a complicated series of experiments to enable us to fix positively the causes of the discrepancies. In their general features the model and full-scale curves agree very well. It should be pointed out also that, although a difference of say, eight per cent. of C_P looks large on a diagram, for practical purposes it is not of primary importance. For constant pitch ratio, revolutions and speed, the diameter of the proper propeller varies as the sixth root of C_P, so that a discrepancy of eight per cent. in the value of C_P means a discrepancy of only about one per cent. in propeller diameter. This is an approximation adequate for the purposes of the engineer. Full-scale tests, where torque and thrust are determined by measurement instead of inference, are of course very desirable, but as far as they go, those to which I have invited

your attention are encouraging to those of us who believe that model experiment properly interpreted is not only valuable but indispensable to aeronautical development.

At the conclusion of the paper Sir SEFTON BRANCKER moved a vote of thanks to Admiral Taylor and to his deputy, Commander Hunsaker, who had so ably

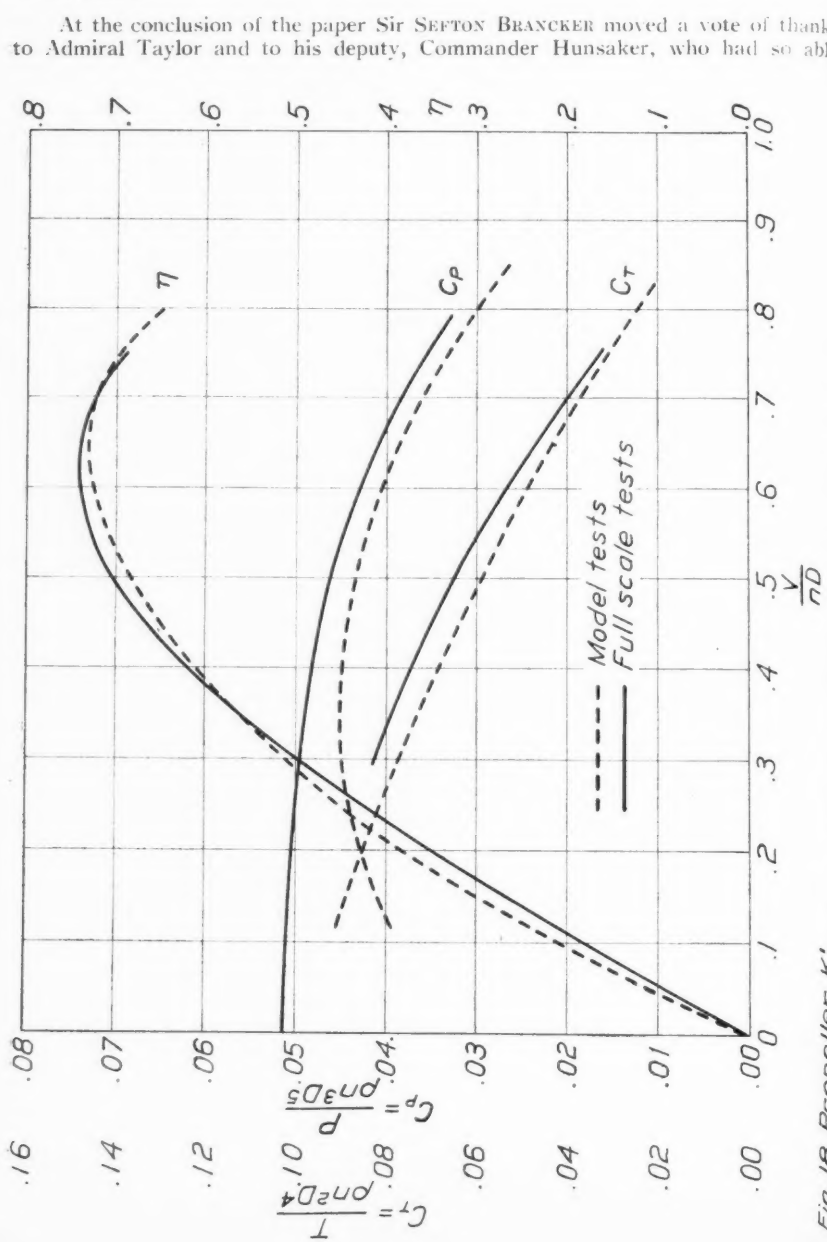


Fig. 18 Propeller K'

abstracted and read it. This particular Wilbur Wright lecture marked the 21st year since the Wrights' first achievement of human flight, and that cordial spirit of co-operation in the exchange of scientific data concerning aeronautics between America and this country, of which this paper was one manifestation, was welcome not merely on its own account but as one item among many making for sound friendship and understanding between the two countries which should be of the

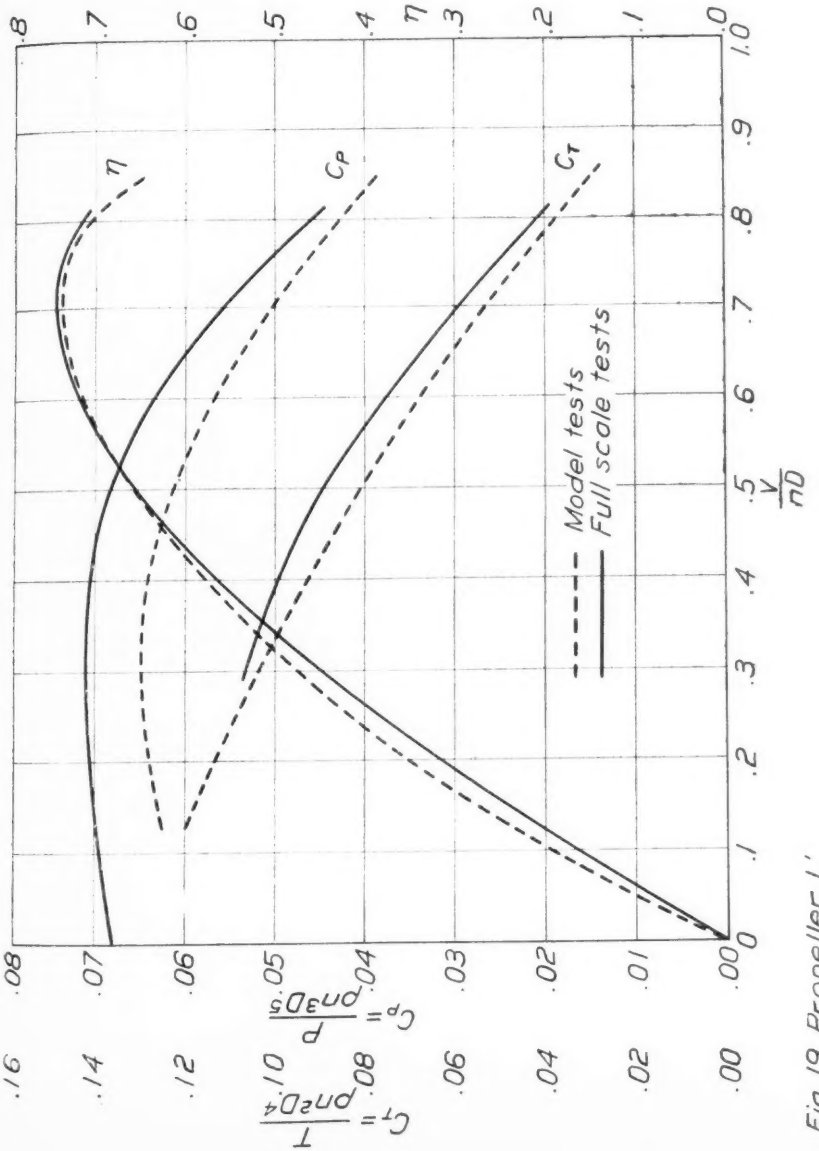


Fig. 19 Propeller L'

utmost importance to the world. Such a friendship would make for the prevalence of peace and honest dealing throughout the world.

Lieutenant-Colonel M. O'Gorman, in seconding the vote, referred to the work of the American National Advisory Committee, of which Admiral Taylor was so prominent a member, and to the way in which their technical reports were freely distributed in this country.

The meeting terminated by a second vote of thanks, moved by Colonel O'Gorman and carried by acclamation, to Lieutenant-Colonel W. Lockwood Marsh for his past services to the Society on the occasion of his last appearance in the official capacity of Secretary.

Colonel Marsh, in reply, expressed his regret at the circumstances which had led to his severing his official connection with the Royal Aeronautical Society.

CORRESPONDENCE

To the Editor of the Journal of the Royal Aeronautical Society.

Sir,—With reference to the extremely interesting work of Dr. Heald and his colleagues (as reported in the June number), may I make a suggestion?

Cannot arrangements be made whereby anyone, who desires to join a light plane club as a flying pupil, can be put through the tests described by Dr. Heald in his lecture for a small fee (say £1)? This would have many and far-reaching advantages. It would enable the clubs to encourage probable "super-pilots" by remitting a part of their tuition fees; to discourage probable "crashers" by increasing their fees (if, against advice, they were determined to try to fly), say 50 per cent. It would thereby save the Air Ministry and/or the clubs much money by reducing the number of crashes. It would relieve club officials of much responsibility in the matter of deciding where to draw the line in admitting persons as flying members. It is in the national interest to encourage all who are keen to take up the sport of flying, even if they are not all likely to be employed as service pilots in an emergency. But a line will no doubt have to be drawn somewhere. It should not be permitted to be drawn on a basis of prejudice or of red tape (as by excluding all women or all persons over forty-five, say, without taking account of their individual qualifications); but persons likely to be looked upon askance by flying club officials would undoubtedly strengthen their positions if they could produce a good, or even a fair average, report from the testing laboratory. On the other hand, if certified to be likely crashers, they could save themselves and others time, money and annoyance by refraining from attempting to learn to fly.

Yours faithfully,

Annie D. Betts.

Camberley.

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